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Sudden area expansion in ducts with flow – A comparison between cylindrical and rectangular modelling

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ABSTRACT

The acoustic properties of an area expansion are analyzed for frequencies where flow acoustic interaction may have a significant influence due to flow separation and vortex shedding. It is investigated why this interaction, which is seen in experimental data on a cylindrical duct as a resonance at a particular Strouhal number of order one, is present in rectangular but not in cylindrical modelling that would be expected to be more realistic; both models consider a plug flow. An analytic method that is suitable for identifying possible reasons for the discrepancies between the two geometries is used. The previously published rectangular model is generalized to the cylindrical case and both models are used to simulate results for all elements in the plane wave scattering matrix and for all parameters for which experimental results are available. The comparison between the two models and between models and measured data is thus not restricted to the flow acoustic induced resonance. The results show that the two geometries in general perform equally when compared with the experimental results, but that the rectangular modelling indeed performs better for some cases. This occurs around a critical Strouhal number, and for higher Mach number. Using the analytic form of the solution, it is shown that the observed discrepancy is related to interaction between the damped hydrodynamic mode and a downstream propagating higher order acoustic mode. Such interaction is not present in the corresponding quiescent duct, and is related to the presence of the shear layer. The analysis shows that the structure of the higher order acoustic modes is different for the cylindrical and rectangular case, respectively, causing the difference in resonant behaviour.

1. Introduction

The modelling of the acoustic properties of a flow duct area expansion and the investigations of the properties of such an element has gained considerable interest in the last decade. Parallel to the development of numerical tools for simulation of wave propagation in flow ducts, the need for benchmark methods and models providing physical insight is increasing. This has resulted in a renewed interest in analytical or semi–analytical models, such as those based on the Wiener-Hopf technique [1–5].

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Modelling of acoustic wave propagation in duct and pipe systems with flow is an important part in the prediction of radiated noise and choice of relevant noise abatement. At sections in the flow duct system where flow separation and vortex shedding occurs, the flow acoustic interaction may have a significant influence on the propagation properties. As the need for efficient numerical simulations increases, it is of interest to investigate how these phenomena can be simulated such that the significant effects of the flow on the acoustics are included. One way to investigate this is through the use of analytical or semi-analytical models.

Several models for two dimensional (rectangular) geometries have been presented for scattering at a sudden area expansion in a flow duct. A natural course to improve the modelling would be to go to cylindrical geometries. However, for numerical flow simulations, the cylindrical computations are often very costly in terms of time, so it is of interest to investigate the improvement that can be gained by the cylindrical modelling. For the understanding of sound propagation in flow ducts, it is also interesting to know more about when the geometry, e.g., rectangular or cylindrical duct cross sections, start to significantly influence the scattering coefficients when we are still in the plane wave regime. A review of the early works on the scattering of acoustic plane waves at a sudden area expansion in a flow duct is found in [2]. Results for the plane wave scattering coefficients in a flow duct have been presented by, e.g., by Boij and Nilsson [2,3], Boij [6] and Kierkegaard et al. [7,8] for a rectangular geometry, by Aurégan et al. [9] for a cylindrical geometry and by Kooijman et al [10,11] for both a cylindrical and a rectangular geometry. The influences of hydrodynamic coupling, flow profile and flow expansion downstream of the expansion have also been studied in these earlier works. Results have been compared with experimental data found in [12] obtained for a cylindrical geometry.

In order to compare the prediction for a rectangular geometry to experimental data for the corresponding cylindrical configuration, a frequency normalization is applied, as proposed by Boij & Nilsson [2]. This normalization, which is described later in this paper, was further evaluated by Kooijman et al. for low frequencies. In that frequency regime, the normalization gives very good correspondence between a rectangular and a cylindrical model except at a critical Strouhal number, defined as

$$St = \frac{ka}{M}, \tag{1}$$

where ka is the Helmholtz number with $k = \omega/c$, where c is the speed of sound and a is the typical cross dimension of the duct, and M is the Mach number defined as $M=U/c$ where U is the mean flow speed in the upstream duct.

Modelling by means of a cylindrical geometry with an axisymmetric sound field was proposed by Kooijman et al. [10,11], who also presented results for the simplified model proposed by Aurégan et al. [9]. Results for low frequencies were compared with results for a rectangular geometry and also with experimental results, for frequencies up to 20% of the cut-on frequency of the larger duct. When comparing the experimental data with the results for the models with rectangular and cylindrical geometry, respectively, it appears that calculations for the cylindrical geometry give less correspondence with experiments than the rectangular does, although this result is not discussed extensively. For the frequency range considered, the discrepancy between modelled results for the two geometries are significantly different mainly for a certain critical Strouhal number range that include a resonance for the rectangular model supported by experiments.

The trend found by Kooijman et al. [10,11] that the rectangular modelling tends to give better agreement with experimental data is unexpected. The results raise several questions: Why is it so? Does it hold also for higher frequencies? (Is it just a coincidence that the rectangular is so good?) In this paper, we investigate the observed discrepancies further. This is done by comparing the results for rectangular and cylindrical geometries for all elements in the scattering matrix using the same plug flow model that Kooijman et al. used and the semi–analytical method proposed by Nilsson and others. This method is suitable for identifying possible reasons behind the discrepancy and for analyzing the mechanisms of the observed resonance. The investigation includes a comparison results for both geometries with the available experimental data for a cylindrical geometry and an extension of the comparison to relatively higher frequencies than was done by Kooijman et al., who went up to $ka=0.23$. A more thorough understanding of this is of interest both for applications and for the fundamental understanding. From a modelling point of view it is interesting to understand how large the influence of the actual geometry, rectangular or cylindrical, is on the acoustic properties. For the industrial applications perspective it is interesting to see if the cylindrical modelling is worthwhile, or if rectangular is enough. Also, it is of interest to further understand how the acoustic properties of a duct area expansion and influence of geometry is changed by the presence of a mean flow.

For a bifurcated duct, the Wiener-Hopf method, with the infinite product splitting procedure in particular, is a powerful method for analyzing scattering properties in physical terms such as modal parameters. In addition, the required edge condition is guaranteed analytically. The modal parameters are determined numerically as well as their combination in the splitting functions. It is therefore appropriate to term the method as semi–analytical. A purely numerical method, using e.g. finite differences or finite elements, cannot provide a corresponding physical interpretation. The Mode Matching Method [13], which is applicable to both bifurcations and sudden area changes, give the solution as a modal series. In contrast to the Wiener-Hopf Method, the modal amplitudes are not simply related to modal parameters but are determined from a relatively large system of linear equations.

It is therefore of interest to generalize the Wiener-Hopf Method for applications to the sudden area change duct. There are primarily two versions for such a procedure. In the first version, The Modified Wiener-Hopf method, the Wiener-Hopf method itself is modified by the addition of an infinite system of equations. In the second version, the Building Block Method or the Cascade Technique, the scattering results from the bifurcation are combined with all multiple reflections caused by the area change by the addition of an infinite linear system of equations. For both versions, the added infinite systems of equations