Journal of Sound and Vibration 394 (2017) 75–89

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Sensitivity analysis of a sound absorption model with correlated inputs

W. Chai, J.-L. Christen, A.-M. Zine, M. Ichchou*

École Centrale de Lyon, Université de Lyon, 36 avenue Guy de Collongue, 69130 Écully, France

ARTICLE INFO

Article history:
Received 20 April 2016
Received in revised form
30 December 2016
Accepted 11 January 2017
Handling Editor: I. Lopez Arteaga
Available online 30 January 2017

Keywords:
Global sensitivity analysis
Correlated inputs
FAST
JCA model
Micro-macro model

ABSTRACT

Sound absorption in porous media is a complex phenomenon, which is usually addressed with homogenized models, depending on macroscopic parameters. Since these parameters emerge from the structure at microscopic scale, they may be correlated. This paper deals with sensitivity analysis methods of a sound absorption model with correlated inputs. Specifically, the Johnson-Champoux-Allard model (JCA) is chosen as the objective model with correlation effects generated by a secondary micro-macro semi-empirical model. To deal with this case, a relatively new sensitivity analysis method Fourier Amplitude Sensitivity Test with Correlation design (FASTC), based on Iman's transform, is taken into application. This method requires a priori information such as variables' marginal distribution functions and their correlation matrix. The results are compared to the Correlation Ratio Method (CRM) for reference and validation. The distribution of the macroscopic variables arising from the microstructure, as well as their correlation matrix are studied. Finally the results of tests shows that the correlation has a very important impact on the results of sensitivity analysis. Assessment of correlation strength among input variables on the sensitivity analysis is also achieved.

1. Introduction

Noise reduction is often a major concern in the industry. Many kinds of porous materials are used for sound absorption, due to their low mass and good performance in medium and high frequencies. Mathematical models such as Johnson-Allard, simplified Lafarge and Champoux-Allard (JCA) model [1] and Biot-Allard model [2] are developed to better estimate the performance of porous material in case of backing by a rigid wall.

As the micro structure of porous material can strongly vary under different fabrication conditions, their physical properties may contain larger uncertainty than classical materials. This explains the growing interest in sensitivity analysis for noise control applications. Specifically, in the study of Ouisse et al. [3] the issue of sensitivity analysis of absorption indicators is dealt with. Christen et al. [4] considered the transmission loss through layered composite panels as an indicator for the sensitivity analysis. In both cases, some interesting trends with regards to the more influential material parameters were provided. *However, in these works, the variables are mostly supposed to be independent and uniformly or normally distributed.* Concerning porous materials, a micro-macro semi-empirical model, recently proposed by Doutres et al. [5], shows that the variables studied in the JCA model may have correlation among them. Other works on acoustic foam's

* Corresponding author.
E-mail address: mohamed.ichchou@ec-lyon.fr (M. Ichchou).

http://dx.doi.org/10.1016/j.jsvi.2017.01.022
0022-460X/

microstructures are also reviewed, such as Zielinski [6], which present similar acoustic properties, while the correlation was not clearly pointed out.

The FAST (Fourier Amplitude Sensitivity Test) sensitivity analysis method used in [3,4], was originally developed by Cukier et al. [7] and Saltelli and Bolado [8] for its high calculation efficiency. This method has also been applied to different cases such as forest planning [9], nuclear waste treatment [10] and system reliability verification [11]. Such implementations proved its effectiveness and robustness in multi disciplinary cases. Several sensitivity analysis methods were developed to handle correlation between variables. Among them, let's mention Kucherenko's advanced Monte-Carlo (MC) estimators [12] which are based on the classic Sobol'-Jensen estimator [13], Mara's sensitivity estimator [14] based on Rosenblatt transformation, and Chastaing's newly developed covariance based sensitivity index [15]. The FAST method with Correlation design (named FASTC in this paper) which is considered further was proposed by Xu and Gertner [16]. The advantage of FASTC is that it requires only the knowledge of correlation matrix, which is more accessible than conditional distribution density function in industrial cases. However, these methods had never, to the authors' knowledge, been applied to vibroacoustic problems.

One of the main objectives of this work is to find out how correlated the variables of the JCA model are, and to estimate the impact of the correlation on SA estimations. This will be achieved through the micro-macro model and by observing their effects on the SA results. These statistics may help to better understand how to handle uncertainties in manufacturing and filtering phase for materials with complex microstructures. Also, this study can be regarded as a first test of the FASTC method's effectiveness and reliability on vibroacoustic applications.

This paper is structured as follows. In Section 2 some basic sensitivity analysis notations are given and the applied FASTC and CRM method are reminded. The JCA porous material model, along with the micro-macro model which generates the correlation, will be detailed in Section 3. Section 4 deals with the results of FASTC in several cases and a comparison with the CRM method. Finally, the limits of the approach based on Iman's transform are discussed in Section 5.

2. Sensitivity analysis methods with correlation design

Sensitivity analysis is *the study of how uncertainty in the output of a model (numerical or otherwise) can be apportioned to different sources of uncertainty in the model input* [22], and several methods have been developed in the last 50 years. Among them some widely used ones, including FAST, belong to the ANOVA (ANalysis Of VAriance) class.

ANOVA denotes a group of SA methods based on a same system of sensitivity indexes S. For a model $Y = f(x_1, x_2, \ldots, x_n)$ with $f: \mathbb{R}^n \to \mathbb{R}$, it has been proved by Sobol' [17] that the total variance of the output $V(Y)$ can be uniquely decomposed into the sum of conditional variances as following under several conditions:

$$V(Y) = \sum_i V_i(x_i) + \sum_i \sum_{j>i} V_{ij}(x_i, x_j) + \sum_i \sum_{j>i} \sum_{l>j} V_{ijl}(\ldots) + \cdots + V_{123\ldots n}(x_1, \ldots, x_n), \tag{1}$$

and the sensitivity indexes are defined as $S_u = V_u/V(Y)$, $u \subseteq \{1, \ldots, n\}$.

A more compact definition of the first order sensitivity index is given by:

$$S_i = \frac{V_{X_i}(E_{X_{\sim i}}(Y|X_i))}{V(Y)}, \tag{2}$$

where $X_{\sim i}$ means all the inputs except X_i. The index S_i represents the ratio of variance of the output Y explained by the input X_i. For systems with uncorrelated inputs, $\sum S_i \le 1$ is always true. When $\sum S_i = 1$, the system is called an additive system. As an important assumption, the whole structure of ANOVA is based on uncorrelated variables, but this is not the common case in industrial applications. There are several reasons why variables become correlated, while the most important one is that these variables themselves are outputs of another model. There are several classical measures of correlation between variables. Among these, one can cite Pearson's and Spearman's coefficients. Pearson's correlation coefficient ρ^p uses the covariance function $\mathrm{cov}(x_i, x_j)$, and is defined as:

$$\rho_{ij}^p = \frac{\mathrm{cov}(x_i, x_j)}{\sigma_{x_i}\sigma_{x_j}}, \tag{3}$$

where σ_x is the standard deviation of x. This coefficient measures the linear dependence between two variables.

Spearman's correlation coefficient is defined as:

$$\rho_{ij}^s = 1 - \frac{6\sum_{k=1}^N (\mathrm{rank}(x_i^k) - \mathrm{rank}(x_j^k))^2}{N(N^2-1)}, \tag{4}$$

where N is the number of samples and $\mathrm{rank}(x)$ is an operator to get the position of each element after sorting them in increasing order. Spearman's correlation coefficient indicates the monotonic relationship between variables.

There are some similarities between these two correlation coefficients. Both of them have values between -1 and 1, where -1 means strictly negative correlation, 1 means strictly positive correlation and 0 means uncorrelated. They both have the properties that $\rho_{ij} = \rho_{ji}$ and $\rho_{ii} = 1$. In most industrial cases, the correlation matrix, which is composed of these