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Mode shape expansions for the dynamic testing of cable domes considering random pretension deviations

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ABSTRACT

Cable domes maintain their structural stability and deformation resistance substantially depending on the geometrical stiffness contributed by pretension. Dynamic testing can be employed to monitor the possible stiffness degeneration caused by pretension deviations in existing cable domes. The measured incomplete mode shapes should be expanded for effectively evaluating the actual structural stiffness. However, conventional methods lose effectiveness for expanding mode shapes of cable domes whose modes are sensitive to the pretension deviations. A novel method is developed in this paper to expand the incomplete mode shapes of existing cable domes with random pretension deviations. For a monitored target mode of the existing structure, its mode shape can be approximately expressed as a linear combination of a few mode shapes of the ideal structure. Once their combinational coefficients are determined based on the measured incomplete mode shape, the expansion of this target mode is achieved. Two key steps are included: the determination of these so-called contribution modes and the estimation of their combinational coefficients. For the prescribed limit values of equivalent member length errors adopted to simulate random pretension deviations, contribution modes can be determined by considering the mode shape variations and mode jumpings. A proposed contribution mode effective independence (CMEI) method is further put forward to obtain the best estimate of combinational coefficients and the optimal layout of sensors. The numerical example of a cable dome illustrates the invalidation of the conventional expansion methods when random pretension deviations are considered. In contrast, the method proposed in this paper is validated to be effective and reliable even in the cases of severe modal variations and high noise levels.

1. Introduction

In recent decades, cable domes have been erected in many landmark long-span buildings worldwide [1, 2] and discussed in numerous research papers [3–5]. Because of its kinematically indeterminate geometry [6], a cable dome maintains the structural stability and deformation resistance substantially depending on the geometrical stiffness contributed by pretension [7, 8]. Actually, pretension deviation is inevitable for an existing cable dome and can be caused by many factors such as the manufacturing error of member lengths, installation deviation of anchoring joints, relaxation of cables, and degeneration of boundary constraints [9]. Structural stiffness is therefore the key point in monitoring an existing cable dome due

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to its possible degradation caused by pretension deviation. At present, the pretension deviation is normally monitored by directly testing the tensile forces of cables. However, the number of tested cables is usually limited because the force sensors (e.g., the magnetic flux sensors [10]) are normally expensive and nonreusable. The structural stiffness of an existing cable dome is therefore difficult to evaluate effectively based on the tested pretensions of very limited cables.

Dynamic testing [11] can be employed to monitor the change of structural stiffness by testing the modal eigenvalues and mode shapes of an existing structure. The structural stiffness can even be reconstructed using these obtained modal properties. Because the sensors, e.g., accelerometers, can be placed flexibly and reused, dynamic testing is therefore a more economical and feasible method for monitoring the structural stiffness of an existing cable dome. Although the dynamic testing method has been developed for a long time, its applications in the monitoring of existing cable domes have been rarely reported.

There are many important issues in the dynamic testing of an existing structure, including the determination of target modes to be monitored, optimal layout of sensors [12], and modal identification [13]. Normally, only incomplete mode shapes of target modes can be obtained due to the limited number of sensors. For evaluating the structural stiffness more accurately, these incomplete mode shapes must be expanded to the complete mode shapes with full structural degrees of freedom (DOFs). This is the main topic of this paper.

Conventional mode shape expansion methods can generally be divided into two types according to the differences of the transformation matrix between the incomplete mode shape and its expanded complete mode shape. The transformation matrices in the first type of methods are directly related to the structural stiffness and/or mass matrices, such as the static (Guyan) expansion method [14] and the dynamic expansion method [15]. For the second type of methods, typical of the system equivalent reduction expansion process (SEREP) method [16], the perturbed force approach [17] and the rapid direct-mode shape expansion method [18], the complete mode shape is described as a linear combination of ideal mode shapes [16,17], or the unmeasured part is linearly combined by hybrid vectors [18]. The transformation matrices are thus indirectly associated with the combinational coefficients which actually imply the variations of the structural stiffness and mass. In these conventional methods, the transformation matrices are normally obtained based on the numerical (ideal) structural model established for structural design, although some also contain measured modal data [17,18]. It should be noted that a cable dome generally presents dense and coincident modal eigenvalues. Its modal properties are actually very sensitive to the changes of structural stiffness caused by pretension deviations. In other words, the validity of mode shape expansions using these conventional methods should be intensively investigated because a distinct difference between the estimated transformation matrices and the actual transformation matrix may exist.

In this paper, a novel method is developed for expanding the incomplete mode shapes of existing cable domes with random pretension deviations. For a monitored target mode, its mode shape can be approximately expressed as a linear combination of a certain number of mode shapes of the ideal structure regardless of pretension deviations. As long as their combinational coefficients can be determined based on the measured incomplete mode shape, the mode shape expansion of this target mode is achieved. The equivalent member length errors are adopted to simulate the random pretension deviations in the existing structure, and the sensitivity relationships between member length errors and modal eigenvalues, as well as mode shapes, are established. For the prescribed limit values of member length errors, an approach for determining the so-called contribution modes is suggested by considering the mode shape variations and mode jumpings [19]. For those contribution modes, a novel method is further put forward to obtain the best estimate of combinational coefficients and the optimal layout of sensors. An illustrative cable dome is employed to investigate the accuracy and validity of the mode shape expansion method proposed in this paper.

2. Conventional mode shape expansion methods

The dynamic characteristic equation of a cable dome can be expressed as

$$\left(\mathbf{K}_{\mathrm{T}} - \eta_j \mathbf{M}\right)\boldsymbol{\theta}_j = \mathbf{0} \tag{1}$$

where $\mathbf{K}_{\mathrm{T}}$ is the tangential stiffness matrix corresponding to the equilibrium configuration to be investigated; $\mathbf{M}$ is the mass matrix; and η_j and $\boldsymbol{\theta}_j$ are the eigenvalue and mass-normalized mode shape of mode j, respectively. The eigenvalue of a target mode can be determined by dynamic testing; however, only its incomplete mode shape can be obtained because the number of sensors is normally limited. Partitioning the matrices and vector corresponding to the measured and unmeasured DOFs, Eq. (1) can be rewritten as

$$\left(\begin{bmatrix} \mathbf{K}_{\mathrm{T}}^{\mathrm{aa}} & \mathbf{K}_{\mathrm{T}}^{\mathrm{au}} \\ \mathbf{K}_{\mathrm{T}}^{\mathrm{ua}} & \mathbf{K}_{\mathrm{T}}^{\mathrm{uu}} \end{bmatrix} - \eta_j \begin{bmatrix} \mathbf{M}^{\mathrm{aa}} & \mathbf{M}^{\mathrm{au}} \\ \mathbf{M}^{\mathrm{ua}} & \mathbf{M}^{\mathrm{uu}} \end{bmatrix}\right)\begin{Bmatrix} \boldsymbol{\theta}_j^{\mathrm{a}} \\ \boldsymbol{\theta}_j^{\mathrm{u}} \end{Bmatrix} = \mathbf{0} \tag{2}$$

where the superscripts a and u denote the measured and unmeasured set of DOFs, respectively. Generally, the incomplete mode shape $\boldsymbol{\theta}_j^{\mathrm{a}}$ is expanded to the complete mode shape $\boldsymbol{\theta}_j$ by the following relationship:

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