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Experimental study of the mapping relationship based near-field acoustic holography with spherical fundamental solutions

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ABSTRACT

This paper is a consequent work of the previously proposed mapping relationship based near-field acoustic holography (MRS-based NAH), [H.J.Wu W.K. Jiang and H.B. Zhang, JSV, 373:66-88, 2016]. It is devoted to the performance study of its practical application with error analysis and experimental validation. Two types of errors, the truncation errors due to the limited number of participant modes, and the inevitable measurement errors caused by uncertainties in the experiment, are considered in the analysis. The influences of the errors on the performance of MRS-based NAH are systematically investigated. First of all, expression of the relative reconstruction error of the pressure energy is derived based on the two types of errors. An approach is developed to estimate the lower and upper bounds of the relative error. It gives a guide to predict the error for a reconstruction under the condition that the truncation error and the signal-to-noise ratio are given. Then, the condition number of the inverse operator is investigated to measure the sensitivity of the reconstruction to the input errors. Asymptotic expressions of the condition number for a special case, conformal spherical model and hologram, are obtained, which indicates the condition number has a geometric growth with the number of participant modes. Numerical examples with different kinds of errors are elaborately designed to validate the stability as well as the correctness of the error analysis. At last, the MRS-based NAH is further examined and verified by a physical experiment, a vibrating cubic model reconstructed from measurement on a spherical hologram. A satisfied agreement with the directly measured pressure on a validation surface is observed for both quantity and distribution of the reconstructed pressure.

1. Introduction

Near-field acoustic holography (NAH) is an effective tool to reconstruct the interested acoustic quantities (e.g. sound pressure, particle velocity) based on a number of measurements at a close distance to a structure which are typically returned by an array microphones or probes. As is well known, it is usually claimed as an ill-posed problem in the sense of Hadamard [1], i.e. it may have no solution at all or the solution may not be unique and it may be extremely sensitive to slight errors in the input. It is due to that the inverse operator is ill-conditioned, e.g. subject to larger condition number, which are

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generally caused by the over selected basises in either numerical or analytical form for the description of acoustic quantities on the surface of structure or hologram. Williams ascribed the root of the ill-posed nature to the rapidly decayed evanescent waves in the radiated pressure [2].

To circumvent this issue, regularization methods were introduced to the NAH. The prevailing approach goes to the popular Tiknonov regularization which controls the solution by balancing the fidelity term and the regularization errors [1,3–6]. However, determination of the amount of regularization parameter is not straightforward, and is a crucial aspect of the successful application of the Tiknonov regularization. Thus, several strategies had been developed to properly determine the regularization parameter, such as generalized cross validation (GCV) [7] and the L-curve [8], etc. However, at present, there is still no absolutely universal method that is robust enough and always produces a good regularization parameter. Truncated singular value decomposition method is another type of popular methods [9,10], which also plays a crucial role in the Tiknonov regularization. Of course, there are some other variant regularization methods, such as recently developed Bayesian regularization method [11,12], etc. However, in a general conclusion, there is no holy grail as to the best regularization approach [2], also pointed in the Ref. [12] that the regularization methods are very problem-dependent and no consensus on which one is the best.

The regularization methods [1,3–6,9–12] were introduced and studied to obtain the reconstructed acoustic quantities as best as possible from the contaminated measurement. However, sometimes the very ill-posed inverse operator makes the regularization a real difficulty, and in turn is very urgent to need a proper regularization method. That is the reason why the regularization method for NAH is still an open question attracting many research works. The ill-poseness is even more serious for incomplete measurement regardless the types of NAH adopted, such as the attempts to reconstruct the acoustic quantities on a three dimensional structure from one or several non-enclosing holograms. While the exact solution is possible to be obtained for the inverse operation from a measurement on a complete hologram enclosing the vibrating structure [12,13]. It is the measurement that can form a unique one-to-one mapping relationship between the surface of structure and hologram [14]. Based on the guideline for determining the number of participant modes as well as the number and positions of microphones, a linearly well-conditioned deterministic system is built for reconstruction. The determination process could be viewed as a pre-regularization in the MRS-based NAH. Thus, the general regularization methods [1,3–6,9–12] are not suggested to the MRS-based NAH.

As errors are inevitable in the practical measurement, it is curious to know how the errors go through the inverse operation and what influence imposed on the accuracy of the reconstruction results. To the best knowledge of authors, few works are devoted to the error analysis of the NAH by comparing with that for the regularization methods. It is because the NAH was usually viewed as a very ill-posed inverse problem for which obtaining a regularized solution is the primary task. Thus, it is difficult to predict or estimate the reconstruction accuracy. Instead of a predictable way, numerical simulation and experimental validation are two frequently adopted methods to investigate the performance of NAH for different parameters [15–17]. For practical problems, it is hard to estimate the accuracy of the reconstructed results. Thus, one aim of the current work is to develop an approach which can predict the reconstructed accuracy for a specific setup of the MRS-based NAH.

This paper is organized as follows: the underlying theory, determination of the necessary number of participant modes as well as the setup guideline of microphone array of the MRS-based NAH are briefly reviewed in Section 2. Error analysis is conducted in Section 3. The relative error as well as its bounds of the reconstructed pressure energy are firstly derived in Section 3.1 without the consideration of truncation error. Consequently, a modified relative error is developed in Section 3.1 which is a more general expression by counting the truncation error in. In addition, condition numbers of translators in the MRS-based NAH are investigated in Section 3.3 to measure the sensitivity of the reconstruction to the change in inputs. Numerical examples are elaborately setup in Section 4 to validate the correctness of the error analysis, and demonstrate the performance the MRS-based NAH including the measurement errors. Section 4.1 is for the error analysis without truncation error, and Section 4.2 is for the more general case with a model which is going to be experimentally studied. An experiment is set up in Section 5 to validate the method for a practical problem. Section 6 concludes the paper with some discussions and remarks of our work.

2. Procedure of the mapping relationship based NAH

2.1. The theory

Assume that the fluid is homogenous, inviscid, compressible and only undergoes small translation movement. The time harmonic sound pressure radiated from a vibrating structure into an infinite domain Ω is described by the well-known Helmholtz equation

$$\nabla^2 p(\mathbf{x}) + k^2 p(\mathbf{x}) = 0 \text{ for } \mathbf{x} \in \Omega \tag{1}$$

where k is the wave number, relating to the acoustic speed c and angular frequency ω by $k = \omega/c$, and $\mathbf{x}$ is a point in the domain. The time component is assumed to be $e^{-i\omega t}$. An alternative expression of Eq. (1) is the boundary integral equation (BIE) [18]