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Dynamic response of geometrically nonlinear, elastic rectangular plates under a moving mass loading by inclusion of all inertial components

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ABSTRACT

Dynamic deformations of beams and plates under moving objects have extensively been studied in the past. In this work, the dynamic response of geometrically nonlinear rectangular elastic plates subjected to moving mass loading is numerically investigated. A rectangular von Karman plate with various boundary conditions is modeled using specifically developed geometrically nonlinear plate elements. In the available finite element (FE) codes the only way to distinguish between moving masses from moving loads is to model the moving mass as a separate entity. However, these procedures still do not guarantee the inclusion of all inertial effects associated with the moving mass. In a prepared finite element code, the plate elements are developed using the conventional nonlinear methods, i.e., Total Lagrangian technique, but all inertial components associated with the travelling mass are taken into account. Since inertial components affect the mass, damping, and stiffness matrices of the system as the moving mass traverses the plate, appropriate time increments shall be selected to avoid numerical instability. The dynamic response of the plate induced by the moving mass is evaluated and compared to previous studies. Also, unlike the existing FE programs, the different inertial components of the normal contact force between the moving mass and the plate are computed separately to substantiate the no-separation assumption made for the moving mass. Also, it is observed that for large moving mass velocities, the peak plate deformation occurs somewhere away from the plate center point. Under the two extreme in-plane boundary conditions considered in this study, it is shown that if the geometrical nonlinearity of plate is accounted for, the deformations obtained would be less than the case with classical linear plate theory.

1. Introduction

Evaluation of the dynamic response of train bridges was the starting point for many researchers and engineers to investigate the moving mass problem, whether in a single or series form, on beams and plates. Various unknowns such as the maximum dynamic displacement of such structures under moving mass and its relevant aspects has attracted the attention

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of many researchers in the last few decades. Earlier studies were carried out using beam elements as the structural members supporting the moving mass. Stokes [1] provided an approximate solution to the problem by ignoring some important parameters such as the mass effect. Similarly, other earlier studies were generally focused on moving load problems in which the moving load inertial effects were restricted only to the gravitational weight force. An extensive investigation on the moving loads with different discrete and continuous patterns traversing uniform and non-uniform beams and plate members was performed by Fryba [2]. He used integral transformations such as Laplace operator to solve the governing dynamic equations of motion, considering the travelling mass merely as a moving load. Lee [3] applied Timoshenko beam theory to study the dynamic response of beams supported by elastic foundation, subjected to single lateral moving load. Dugush and Eisenberg [4] used modal superposition method to study the dynamic vibration of non-uniform Euler-Bernoulli beams supporting moving loads. Horizontally curved beams subjected to vertical and horizontal moving loads were analyzed by Yang and Wu [5], using appropriate in and out of plane spatial shape functions.

While the case of travelling loads over beams is still under intensive study, an increasing number of researchers have addressed the effect of all inertial components associated with the moving masses. It was shown that the effect of other inertial terms are notably significant especially when the weight of the moving mass becomes considerable in comparison with the mass of the main structural member [6,7]. In this regard, Nikkhoo et al. [7] included all inertial effects to study the dynamic behavior of Euler-Bernoulli beams under moving masses. Moreover, they used classical linear optimal control method to suppress the response of the dynamic system.

On the other hand, the problem of moving masses over structural plate members have been treated by different researchers, in which, the classical linear plate theory known as Kirchhoff theory were employed to describe the dynamic behavior of plates. Rofooei and Nikkhoo [8] investigated the center point dynamic response of thin linear classical rectangular plates under moving mass with a rectilinear path and inclusion of all inertial components of accelerations. Applying modal superposition method, they studied the efficiency of piezoelectric patches utilized as actuators to reduce the dynamic vibrations caused by the moving mass. Meanwhile, other researchers applied more accurate plate theories to estimate the dynamic response of plates. Malekzadeh et al. [9] used cross-ply laminated thick plates in his study. Ignoring non-gravity inertial effects, he merely worked on moving load problem. Shadnam et al. [10] studied dynamic responses of geometrically thin plates by a single differential equation. However, they ignored the convective inertial components associated with the moving mass. Vaseghi Amiri et al. [11] studied the Mindlin plates subjected to a traversing moving mass. They used eigenfunction expansion method with time dependent amplitude factors to evaluate the effect of shear deformations on the dynamic response of the plate.

Obviously, due to great capabilities of finite element techniques, they have been widely used from the earliest studies on the moving mass problem [2]. Martinez et al. [12] have proposed a combined finite element and analytical method to obtain dynamic response of non-uniform, Euler-Bernoulli beams crossed by a discrete moving load. The resources on the finite element modeling and analysis of linear and nonlinear plates are really extensive [13–16]. This technique has also been accounted for non-linear plate theories in numerous investigations [17–20]. Though some studies have discussed moving masses travelling over geometrically nonlinear plates, inclusion of all inertial components related to the moving mass in finite element formulation, requires more efforts. In case of a normal force moving on a plate, the conventional FE programs can convert the applied moving load into equivalent nodal forces, even if the concentrated load is not positioned on any of the mesh nodes. But, if the concentrated vertical load is replaced by a moving mass, the resulting inertial effect would be lost unless it is placed on the nodes of the plate elements.

Therefore, the mesh size used for the plate is restricted by the time increment required for dynamic analysis as well as the moving mass trajectory. Since the problem cannot be solved with required accuracy unless a very fine mesh is used for the plate medium, the dynamic analyses would become extremely time consuming. Even in that case, only the vertical translational inertia effect of the moving mass would be taken into account, and the effect of all other inertial components would be lost. This is owing to the nature of Contact-Target elements used in FE codes to establish the interaction between the moving mass (contact element) and the main plate member (target element). These elements fulfill the equilibrium relations by exerting reaction forces on each other and cause no changes to the initial damping and stiffness of the plate elements, while in real cases, convection inertial components of the moving mass add some time-dependent terms to the total damping and stiffness matrices of the system. Also, applying the available FE programs, it is not possible to decompose the normal contact force existing between the plate elements and the moving mass, into its components which are related to different inertial terms associated with the moving mass.

In this work, appropriate procedures are introduced and combined with the available FE techniques. In this regard, the total Lagrangian (T.L.) technique is employed to prepare a finite element formulation for a geometrically nonlinear plate based on von Karman assumptions. Required relations and procedures to prepare nonlinear dynamic FE codes in MATLAB are previously well-discussed in the literature [13–16] and applied in this study. Even though the constant acceleration method, which is an unconditionally stable technique is used [21], the arbitrary selection of time increment may lead to spectral radius greater than one. This issue has not been considered in most previous works dealing with moving mass problem.

Furthermore, the no-separation assumption is verified by evaluating the different components of the normal force which are related to pseudo-mass, pseudo-damping and pseudo-stiffness effects of the moving mass. Numerical examples are used to demonstrate the performance of the prepared nonlinear FE codes. Also, it is demonstrated that inertial acceleration parameters of the moving mass cannot be neglected for a wide range of mass velocities and weights. Although, the prepared