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Parametric analysis of the statistical model of the stick-slip process

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ABSTRACT

In this paper it is performed a parametric analysis of the statistical model of the response of a dry-friction oscillator. The oscillator is a spring-mass system which moves over a base with a rough surface. Due to this roughness, the mass is subject to a dry-frictional force modeled as a Coulomb friction. The system is stochastically excited by an imposed bang-bang base motion. The base velocity is modeled by a Poisson process for which a probabilistic model is fully specified. The excitation induces in the system stochastic stick-slip oscillations. The system response is composed by a random sequence alternating stick and slip-modes. With realizations of the system, a statistical model is constructed for this sequence. In this statistical model, the variables of interest of the sequence are modeled as random variables, as for example, the number of time intervals in which stick or slip occur, the instants at which they begin, and their duration. Samples of the system response are computed by integration of the dynamic equation of the system using independent samples of the base motion. Statistics and histograms of the random variables which characterize the stick-slip process are estimated for the generated samples. The objective of the paper is to analyze how these estimated statistics and histograms vary with the system parameters, i.e., to make a parametric analysis of the statistical model of the stick-slip process.

1. Introduction

Structural vibration induced by dry-friction is widespread and appears in several applications. In some systems, the vibration generated by dry-friction is primarily responsible for the system functionality. Friction dampers in gas turbines, sound produced by violin strings, and disk brakes [1] are examples of situations in which the achievement of the system targets depends on the existence of dry-friction.

In other systems, the vibration generated by dry-friction disturbs the system functionality. A classical example of such situation is the drilling process [2,3]. The dry-friction force that exists between the bit and the rock can be big enough to stuck the bit during some time intervals. As a constant velocity is imposed at the top of the drillstring, the drillstring is twisted. In consequence, the drillstring acts as a spring and accumulates energy in terms of torsion up to the instant that the stick end and the drillstring it is suddenly released. This phenomenon generates torsional vibrations and instabilities in the system dynamics [3]. It can be harmful to the drilling process and causes fatigue in the drillstring. Furthermore, it causes

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waste of energy since while the bit is stuck there is no penetration.

Regardless if there is gain or loss due to the presence of friction, to better understand and control systems with dry-friction forces, we need to characterize how the system dynamics is affected by it. We need to understand how the dynamics vary with the system parameters, i.e., need to make a parametric analysis. Researchers working in different domains, such as non-linear dynamics and tribology, are interested in the parametric characterization of the dynamics of systems with friction. The literature dealing with the problem is vast [4–7] and the study comprises several application areas (see for instance [8,9]).

Despite the great number of papers in the area, few of them address the problem with a stochastic approach. The majority of the references that characterizes the dynamics with dry-friction only make it with a deterministic approach. They do not discuss or quantify the uncertainties that are involved in the dynamics, although the dry friction force itself presents an inherent random behavior [10]. The influence of ambient conditions in the properties of contact surfaces [11,12] and the dependency on the relative velocity of the bodies in contact make the dry friction force uncertain. Beyond this, dry-friction appears in mechanical systems in which uncertainties play an important role. For example, in drilling some sources of uncertainties are the bit-rock interaction, the presence of impacts, and the fluid-structure interaction [13,14,2]. Randomness arises also from manufacturing, assembly errors, and random load. Therefore, a stochastic approach is the ideal way to address problems with dry-friction [15–18].

In this paper, we analyze the dynamics of a dry-friction oscillator which moves over a base with a rough surface. The base has an imposed stochastic bang-bang motion which excites the system in a stochastic way. The non-smooth behavior of the dry-friction force [19–22] associated with the non-smooth stochastic base motion induces in the system stochastic stick-slip oscillations. The system response is composed by a random sequence alternating stick and slip-modes. To characterize it, we construct a statistical model, in which the variables of interest of the random sequence are modeled as random variables. Examples of these variables are the number of time intervals in which stick or slip occur, the instants at which they begin, and their duration.

To fully characterize the random sequence, it would be necessary to determine its probabilistic model, i.e., the analytical expression of the joint density functions of all random variables of the statistical model. As this task is almost unattainable and hugely ambitious, we set for less. We estimate statistics and histograms of the random variables which characterize the stick-slip process. With them, we get information about the probability distributions of the random variables, but we do not have their mutual interaction, given by the joint densities. For a large number of samples, a histogram approaches the probability density function [23,24].

To estimate statistics and histograms of the system responses, samples of the random sequence of stick and slip-modes are computed by the integration of the dynamic equation of the system using independent samples of the base motion. The objective of the paper is to analyze how the estimated statistics and histograms vary with the system parameters, i.e., to make a parametric analysis of the statistical model of the stick-slip process. Such analysis is new in the literature.

A parametric analysis of a nonlinear system can be computed by the numerical integration of the dynamic equation of the system for different combinations of the values of the system parameters. In the traditional deterministic parametric analysis, for each combination of the system parameters, a single simulation have to be computed. The obtained responses represent how the system behaves. In a stochastic parametric analysis, for each combination of the system parameters, a large number of integrations have to be computed in order to get samples of the system response. Each realization is just one possibility of system response. From a set with a large number of realizations, statistics and histograms are estimated.

The non-smooth behavior of dry-friction oscillator analyzed in this paper turns the numerical integration of its dynamic very time consuming. As the time response exhibits a sequence of stick and slip-modes, to characterize it, it is necessary to identify with accuracy the instants of switch. Consequently, a very small time-step is required. Besides, the number of switches should be big enough to reveal the sequence behavior. Therefore, a huge number of integration time-step is needed. As we perform a stochastic parametric analysis, in which thousands of integrations have to be computed, the total computational cost is huge. It is important to remark that this cost is much higher then the cost required in the traditional deterministic parametric analysis. Currently, the high computational cost required in simulations of stochastic dynamical systems is one of the main obstacles to the development of the area. To face the problem and to reduce the simulation time, we adopted the strategy of parallelization of the simulations. However, even using a cluster composed of sixteen computers, the computation time necessary to perform the integrations was 55,5 days.

This paper is organized as follows. Section 2 describes the stick-slip oscillator analyzed. A probabilistic model to the base motion is construct in Section 3. The definition of the random variables which characterizes the variables of interest in the stick-slip process is made in Section 4. An explanation about the choice of the parameters considered in the stochastic parametric analysis is given in Section 5. In Section 5 it is also discussed the computational cost required to make the stochastic parametric analysis, and it is presented the strategy adopted to reduce it. The influence of λ in the statistics and histograms is discussed in Section 6, and the influence of μ is discussed in Section 7.

2. Dynamics of the stick-slip oscillator

The system analyzed is composed by a simple oscillator (mass-spring) moving on a rough surface, as shown in Fig. 1. The roughness induces a dry-frictional force between the mass and the base which is modeled as a Coulomb friction.