ARTICLE IN PRESS

Journal of Sound and Vibration ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Mechanical energy and equivalent differential equations of motion for single-degree-of-freedom fractional oscillators

Jian Yuan [a,*], Youan Zhang [b], Jingmao Liu [c], Bao Shi [a], Mingjiu Gai [a], Shujie Yang [a]

[a] Institute of System Science and Mathematics, Naval Aeronautical and Astronautical University, Yantai 264001, PR China
[b] Institute of Technology, Yantai Nanshan University, Yantai 265713, PR China
[c] Shandong Nanshan International Flight Co., LTD, Yantai 265713, PR China

ARTICLE INFO

Article history:
Received 4 October 2016
Received in revised form
17 February 2017
Accepted 23 February 2017
Handling Editor: W. Lacarbonara

Keywords:
Fractional oscillators
Viscoelasticity
Mechanical energy
Mean equivalent viscous damping
Mean equivalent stiffness

ABSTRACT

This paper addresses the total mechanical energy and equivalent differential equation of motion for single degree of freedom fractional oscillators. Based on the energy storage and dissipation properties of the Caputo fractional derivatives, the expression for total mechanical energy in the single degree of freedom fractional oscillators is firstly presented. The energy regeneration due to the external exciting force and the energy loss due to the fractional damping force during the vibratory motion are analyzed. Furthermore, based on the mean energy dissipation and storage in the fractional damping element in steady-state vibration, two new concepts, namely mean equivalent viscous damping and mean equivalent stiffness are suggested and the above coefficient values are evaluated. By this way, the fractional differential equations of motion for single-degree-of-freedom fractional oscillators are equivalently transformed into integer-order ordinary differential equations.
© 2017 Elsevier Ltd All rights reserved.

1. Introduction

Viscoelastic materials and damping treatment techniques have been widely applied in structural vibration control engineering, such as aerospace industry, military industry, mechanical engineering, civil and architectural engineering [1]. Describing the constitutive relations for viscoelastic materials is a top priority to seek for the dynamics of the viscoelastically damped structure and to design vibration control systems.

Recently, the constitutive relations employing fractional derivatives which relate stress and strain in materials, also termed as fractional viscoelastic constitutive relations, have witnessed rapid development. They may be viewed as a natural generalization of the conventional constitutive relations involving integer order derivatives or integrals, and have been proven to be a powerful tool of describing the mechanical properties of the materials. Over the conventional integer order constitutive models, the fractional ones have vast superiority. The first attractive feature is that they are capable of fitting experimental results perfectly and describing mechanical properties accurately in both the frequency and time domain with only three to five empirical parameters [2]. The second is that they are not only compatible with thermodynamics [3–5] and the molecule theory [6], but also represent the fading memory effect [2] and high energy dissipation capacity [7]. Finally, from mathematical perspectives the fractional constitutive equations and the resulting fractional differential equations of vibratory motion are compact and analytic [8].

* Corresponding author.
E-mail address: yuanjianscar@gmail.com (J. Yuan).

http://dx.doi.org/10.1016/j.jsv.2017.02.050
0022-460X/© 2017 Elsevier Ltd All rights reserved.

Please cite this article as: J. Yuan, et al., Mechanical energy and equivalent differential equations of motion for single-degree-of-freedom fractional oscillators, Journal of Sound and Vibration (2017), http://dx.doi.org/10.1016/j.jsv.2017.02.050

Nowadays many types of fractional order constitutive relations have been established via a large number of experiments. The most frequently used models include the fractional Kelvin-Voigt model with three parameters [2]: $\sigma(t) = b_0\varepsilon(t) + b_1\mathrm{D}^\alpha\varepsilon(t)$, the fractional Zener model with four parameters [3]: $\sigma(t) + a\mathrm{D}^\alpha\sigma(t) = b_0\varepsilon(t) + b_1\mathrm{D}^\alpha\varepsilon(t)$, and the fractional Pritz model with five parameters [9]: $\sigma(t) + a\mathrm{D}^\alpha\sigma(t) = b_0\varepsilon + b_1\mathrm{D}^{\alpha_1}\varepsilon(t) + b_2\mathrm{D}^{\alpha_2}\varepsilon(t)$, where $\sigma(t)$ is the stress, $\varepsilon(t)$ is the strain, D^α denotes fractional derivatives with the fractional-order α, coefficients a, b_0, b_1, b_2 are positive constants.

Fractional oscillators, or fractionally damped structures, are systems where the viscoelastic damping forces in governing equations of motion are described by constitutive relations involving fractional order derivatives [10]. The differential equations of motion for the fractional oscillators are fractional differential equations. Researches on fractional oscillators are mainly concentrated on theoretical and numerical analysis of the vibration responses. Investigations on dynamical responses of single-degree-of-freedom (SDOF) linear and nonlinear fractional oscillators, multi-degree-of-freedom (MDOF) fractional oscillators and infinite-degree-of-freedom fractional oscillators have been reviewed in [10]. Asymptotically steady state behavior of fractional oscillators have been studied in [11,12]. Based on the functional analytic approach, the criteria for the existence and the behavior of solutions have been obtained in [13–15], and particularly in which the impulsive response function for the linear SDOF fractional oscillator is derived. The asymptotically steady state response of fractional oscillators with more than one fractional derivatives have been analyzed in [16]. Considering the memory effect and prehistory of fractional oscillators, the history effect or initialization problems for fractionally damped vibration equations has been proposed by Fukunaga [17] and Hartley and Lorenzo [18,19].

Stability synthesis for nonlinear fractional differential equations have received extensive attention in the last five years. Mittag-Leffler stability theorems [20,21] and the indirect Lyapunov approach [22] based on the frequency distributed model are two main techniques to analyze the stability of nonlinear systems, though there is controversy between the above two theories due to state space description and initial conditions for fractional systems [23]. In spite of the increasing interest in stability of fractional differential equations, there's little results on the stability of fractionally damped systems. For the reasons that Lyapunov functions are required to correspond to physical energy and that there exist fractional derivatives in the differential equations of motion for fractionally damped systems, it is a primary task to define the energies stored in fractional operators.

Fractional energy storage and dissipation properties of Riemann-Liouville fractional integrals is defined [24,25] utilizing the infinite state approach. Based on the fractional energies, Lyapunov functions are proposed and stability conditions of fractional systems involving implicit fractional derivatives are derived respectively by the dissipation function [24,25] and the energy balance approach [26,27]. The energy storage properties of fractional integrator and differentiator in fractional circuit systems have been investigated in [28–30]. Particularly in [29], the fractional energy formulation by the infinite-state approach has been validated and the conventional pseudo-energy formulations based on pseudo state variables has been invalidated. Moreover, energy aspects of fractional damping forces described by the fractional derivative of displacement in mechanical elements have been considered in [31,32], in which the effect on the energy input and energy return, as well as the history or initialization effect on energy response has been presented.

On the basis of the recently established fractional energy definitions for fractional operators, our main objective in this paper is to deal with the total mechanical energy of a single degree of freedom fractional oscillator. To this end, we firstly present the mechanical model and the differential equation of motion for the fractional oscillator. Then based on the energy storage and dissipation in fractional operators, we provide the expression of total mechanical energy in the single degree of freedom fractional oscillator. Furthermore, we analyze the energy regeneration due to the external exciting force and the energy loss due to the fractional damping force in the vibration processes. Finally, based on the mean energy dissipation of the fractional damping element in steady-state vibration, we propose a new concept of mean equivalent viscous damping and determine the expression of the damping coefficient.

The rest of the paper is organized as follows: Section 2 retrospect some basic definitions and lemmas about fractional calculus. Section 3 introduces the mechanical model and establishes the differential equation of motion for the single degree of freedom fractional oscillator. Section 4 provides the expression of total mechanical energy for the SDOF fractional oscillator and analyzes the energy regeneration and dissipation in the vibration processes. Section 5 suggests a new concept of mean equivalent viscous damping and evaluates the value of the damping coefficient. Finally, the paper is concluded in Section 6 with perspectives.

2. Preliminaries

Definition 1. The Riemann-Liouville fractional integral for the function $f(t)$ is defined as

$$ {}_a\mathrm{I}_t^\alpha f(t) = \frac{1}{\Gamma(\alpha)}\int_a^t (t-\tau)^{\alpha-1} f(\tau)\mathrm{d}\tau, \tag{1}$$

where $\alpha \in R^+$ is an non-integer order of the factional integral, the subscripts a and t are lower and upper terminals respectively.

Download English Version:

https://daneshyari.com/en/article/4924345

Download Persian Version:

https://daneshyari.com/article/4924345

Daneshyari.com