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An offline approach for output-only Bayesian identification of stochastic nonlinear systems using unscented Kalman filtering

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ABSTRACT

In this paper an offline approach for output-only Bayesian identification of stochastic nonlinear systems is presented. The approach is based on a re-parameterization of the joint posterior distribution of the parameters that define a postulated state-space stochastic model class. In the re-parameterization the state predictive distribution is included, marginalized, and estimated recursively in a state estimation step using an unscented Kalman filter, bypassing state augmentation as required by existing online methods. In applications expectations of functions of the parameters are of interest, which requires the evaluation of potentially high-dimensional integrals; Markov chain Monte Carlo is adopted to sample the posterior distribution and estimate the expectations. The proposed approach is suitable for nonlinear systems subjected to non-stationary inputs whose realization is unknown, and that are modeled as stochastic processes. Numerical verification and experimental validation examples illustrate the effectiveness and advantages of the approach, including: (i) an increased numerical stability with respect to augmented-state unscented Kalman filtering, avoiding divergence of the estimates when the forcing input is unmeasured; (ii) the ability to handle arbitrary prior and posterior distributions. The experimental validation of the approach is conducted using data from a large-scale structure tested on a shake table. It is shown that the approach is robust to inherent modeling errors in the description of the system and forcing input, providing accurate prediction of the dynamic response when the excitation history is unknown.

1. Introduction

System identification (SI) is the process of using measurements of the response of a system to infer the parameters that define its mathematical models [1,2]. Several applications of SI have been presented in structural mechanics, including operational condition assessment and management, improvement of design methods, structural control, and structural reliability applications. For structural and mechanical systems whose dynamic behavior can be captured by linear models an extensive literature of SI algorithms has been developed over the past decades [1,2]. However, in some applications a linear model cannot capture features of the dynamic response of a system of interest. Typical sources of nonlinearity include large displacements, large deformations, material nonlinearity, boundary conditions, energy dissipation devices for vibration suppression, actuators, among others [3]. Identification of nonlinear systems presents a significantly increased challenge with respect to its linear counterpart, mainly because of the lack of a general input-output map for nonlinear operators [4].

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In the identification of dynamic systems it is often the case that within a proposed model class a set of uncertain parameters is consistent with the available data, rather than a single point value. Furthermore, the system may be subjected to forcing inputs that are difficult to measure, as occurs for example in structural systems subjected to wind-induced effects, offshore structures and bridges with submerged foundations. Sensors malfunction is also a motivation for the development of approaches that do not rely on knowledge of the input time history [5]. In such applications a probabilistic or stochastic model can be adopted to quantify and propagate the uncertainties involved in the modeling of dynamic systems [6]. In this context Bayesian inference provides a logically consistent, robust and rigorous theory that can be applied to characterize modeling uncertainty and system identification for a wide class of problems [7–12]. In Bayesian system identification the uncertainty in the parameters is characterized by a probability density function (PDF) conditional in the available data, known as the posterior distribution.

Bayesian SI methods can be broadly classified as online (also known as real-time, recursive or sequential estimation) and offline (also known as batch estimation). Online approaches aim to estimate the parameters sequentially as the data becomes available, while offline approaches consider a fixed observation record. Popular methods for online Bayesian SI include the Kalman filtering based algorithms (extended, unscented and ensemble Kalman filters) [13,14] and stochastic simulation based approaches (particle filters) [15,16]. Among the various Bayesian filters in the literature the unscented Kalman filter (UKF) has received notable attention in structural mechanics applications, mainly due to its increased accuracy with respect to the extended Kalman filter at a less computational cost than sampling based filters (ensemble Kalman and particle filters) [17–23]. Recent work in Bayesian SI in nonlinear systems has focused in (augmented-state) joint state-parameter estimation using online methods in applications where the time history of the main component of the forcing input is known (measured) up to an additive noise [15,24,16,17,25]. Applications where the main component of the input is unmeasured and modeled as a stochastic process are sought herein; the identification under such conditions will be referred to as output-only identification [26]. Effective probabilistic approaches for output-only parameter identification in stationary and/or linear systems have been previously proposed in the literature [26–28]. However, the development of robust and efficient output-only Bayesian SI approaches for non-stationary nonlinear structural and mechanical systems has been limited [5].

In this paper an offline approach for Bayesian identification of stochastic nonlinear systems is presented. In contrast to existing output-only identification approaches that typically assume a white noise input model, non-stationary nonlinear systems are studied herein. In the proposed approach the augmented-state parameter estimation problem is decoupled and performed in two sequential stages: an offline Bayesian SI stage where only model parameters are estimated, and a state estimation stage where model predictions are performed using all plausible models of the model class. For this purpose the parameters posterior PDF is re-parameterized in order to include the state predictive distribution as shown in a further section of the paper; the correlation between the measurements, the state and the parameters is provided by the postulated stochastic model class. The state predictive distribution is recursively estimated using an unscented Kalman filter in a state estimation step, bypassing the state augmentation performed by existing online techniques. The decoupling of the problem renders an algorithm with increased numerical stability. Moreover, the proposed approach has the capability to handle non-Gaussian prior and posterior distributions, in contrast to standard augmented-state unscented Kalman filtering estimation that inherits from Kalman filtering the need to use only second-order statistics. The proposed approach is not suitable for applications where an estimate of the state and the parameters is needed near real-time, such as control applications. Offline Bayesian identification algorithms of this kind have become increasingly popular, mainly because of their enhanced stability and robustness with respect to augmented-state parameter estimation approaches when the dimension of the parameter space is relatively large, the number of response measurements is limited and input time history is unknown [29,30]. In particular it is well-known that for nonlinear systems online algorithms perform poorly in high-dimensional parameter spaces, an issue that is further exacerbated when the forcing input history is not available during the estimation.

In offline Bayesian SI the evaluation of potentially high-dimensional integrals needs to be performed to obtain expectations of functions of the parameters. For example, the parameters marginal and bivariate distributions reveal non-unique optimal estimates and correlations between the parameters. Markov chain Monte Carlo (MCMC) is adopted as an efficient method to sample the joint posterior distribution and approximate the expectations using sampling statistics. In MCMC a Markov chain is constructed such that the stationary distribution of the chain is a target distribution (in this case the parameters posterior distribution). To construct the Markov chain the re-parameterized posterior is used to evaluate the likelihood function using state estimation-based unscented Kalman filtering. The resulting approach will be referred to as the UKF-MCMC approach. Related approaches based on the application of ensemble Kalman and particle filters have been previously proposed in the literature [30]; these approaches have the capability to handle stronger nonlinearities at the expense of a significant increase in the required computational effort. The UKF-MCMC approach uses the unscented Kalman filter to efficiently compute the state predictive distribution, reducing the computational resources demanded by sampling-based stochastic filters.

The proposed UKF-MCMC approach is numerically verified using synthetic data in a bilinear hysteretic oscillator subjected to Gaussian white noise, and a four degree of freedom nonlinear chain subjected to a base acceleration consisting of a modulated Kanai-Tajimi process. The examples show that the approach has the capability to estimate the model parameters correctly, providing confidence intervals to characterize the uncertainty in the estimates. Moreover, the second numerical example is used to numerically illustrate the increased stability of the proposed approach with respect to augmented-state UKF parameter estimation. The enhanced stability is attributed to the improved observability/identifiability conditions of

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