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Dominant modal decomposition method

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ABSTRACT

The paper deals with the automatic decomposition of experimental frequency response functions (FRF's) of mechanical structures. The decomposition of FRF's is based on the Green function representation of free vibratory systems. After the determination of the impulse dynamic subspace, the system matrix is formulated and the poles are calculated directly. By means of the corresponding eigenvectors, the contribution of each element of the impulse dynamic subspace is determined and the sufficient decomposition of the corresponding FRF is carried out. With the presented dominant modal decomposition (DMD) method, the mode shapes, the modal participation vectors and the modal scaling factors are identified using the decomposed FRF's. Analytical example is presented along with experimental case studies taken from machine tool industry.

1. Introduction

The main goal of this work is to develop a modal identification technique in order to extract the dominant dynamic behaviour of a machine tool structure automatically. The need for such an automatic modal identification is mentioned in [1]. The identification technique should be suitable for performing error analysis and stability predictions of certain machining (e.g. milling) operations [2]. The mechanical structures of turning and milling machines are usually stiff (and consequently slightly damped) to withstand continuous and interrupted forcing originated from the cutting processes. Moreover, in case of large heavy-duty machines, the dynamic behaviour of these structures varies along the workspace causing additional difficulties in modelling.

A challenging problem of dynamic characterization of machines is that it requires human interaction with expert knowledge in dynamics, which is fairly expensive and not always available in industry. According to current expectations, future machine tools are envisioned having applied cyber-physical system (CPS) capabilities [3] that give sufficient self-sensing and self-acting functionalities based on preferably non-parametric based techniques.

In most cases, only the dominant vibration modes are needed to predict the stability and the quality of machining processes. Consequently, accurate modelling is not an expectation here. Most of the commercial techniques are not fully automatized; an expert is needed to select the relevant modes, the relevant bandwidth and the correct model-order, also to exclude the malicious modes. Notice, that there already exist well-based methods that are capable of building an exact model of the entire dynamics of a given structure [4–6], however, the aim of the present paper is restricted to the identification of the dominant dynamics, but in an automatic way.

In order to extract modal parameters, complex time domain and frequency domain methods were developed together

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with one-directional, single degree-of-freedom (DOF) mode-picking methods [7–9], frequency domain decomposition FDD [10] and mode indication functions such as MvMIF and CMIF [11]. Of course, a general method is expected to provide acceptable solution in multi-DOF (MDOF), multi-directional, and multi-input/multi-output (MIMO) cases, too. Theoretically, the same information is available both in the time and in the frequency domains, however, time domain methods are usually preferred for slightly damped systems (like machine tool structures), while frequency domain methods are convenient in case of large damping. In the absence of sufficient excitation, techniques related to operational modal analysis should also be mentioned; these determine only the poles in case of large structures [12,13].

The goal of frequency domain methods is to fit directly the analytical expression of the frequency response function (FRF) to the measurements. Consequently, the most straightforward method is to formulate a nonlinear least square (LS) problem (LSFD [14]) that leads to a nonlinear algebraic system of equations, which determines the modal parameters. In order to avoid iterative solution initiated optionally, linear formulation and/or approximation of the LS problem are applied. This can be achieved, for example, by means of the total LS (TLS) method [15], the least-squares complex frequency-domain (LSCF) method [16], or the rational fraction polynomial (RFP) method [17]. The introduction of orthogonal describing polynomials (OP, [18–20]) makes the RFP method well conditioned. In the frequency domain, another solution is the polyreference algorithm (PFD, [21,22]), which utilizes the relation between responses in time and frequency domains (PLSCF, [23–25]). The so-called polyMAX algorithm can be considered as further evolution of the LSCF and the PLSCF methods [26]. Stable LS algorithm is presented in [27], which is able to deal with MIMO systems with minimal user interaction. Automated methodology for PLSCF (polyMAX) is presented in [28] where the authors summarize pole selection method for pole stability charts; case studies are shown in [29]. The statistical basis of the automated selection algorithm is presented in [30].

Time domain methods work with impulse response functions (IRF's) directly, like in case of the least squares complex exponential (LSCE) method [31,32]. This method considers each value of the sampled IRF's as a linear combination of previous values with unknown coefficients. The coefficients are determined by a LS algorithm, from which the poles and the residues can be derived. The polyreference in time domain (PTD) [33,34] takes the analytical expression of the IRF's and determines the poles and the modal participation matrices from a polynomial approximation of a map corresponding to the characteristic equation. The residues are calculated in the second stage of the algorithm by using LS algorithm. In addition, the NEXT approach should be mentioned as an alternative way to determine modal parameters for high modal damping [35] based on the correlation functions of time responses. The stochastic subspace identification SSI techniques [36–38] use the measured time evolution of output state vectors generated by white noise excitation.

Contrary to the previous methods, the Ibrahim time domain method (ITD, [39–41]) and eigenvalue realization algorithms (ERA, [42–44]) calculate the modal parameters directly from the bi-sampled IRF's or FRF's. The method proposed in this study is an extension in this direction with a rather different algorithm, developed specifically for the machine tool industry to deal with weakly damped structures. This algorithm is also based on bi-sampled IRF's and provides the system matrix directly in an alternative simple way by introducing the impulse dynamic subspace (IDS) of the corresponding mechanical system. With the help of the eigenvectors of the system matrix, an appropriate decomposition of the dynamics is possible where one mode is dominant. Afterwards, a convenient technique is introduced to determine modal scaling factors and mode shapes leading to an efficient algorithm that is capable of selecting the dominant dynamics automatically.

In summary, the goal is to present an alternative automatized modal analysis technique as an extension of ERA [45] and LSFD [46,28] techniques. The paper explains the concept of IDS in the continuous time domain, and it also derives the practical sampled discrete case. As a consequence, the system matrix of the corresponding free vibratory system can be derived in an efficient way. The automatic identification of the relevant poles and the exclusion of the malicious IDS' make it possible to construct a dominant decomposition of the FRF's. Throughout the paper, we refer to the proposed technique as dominant modal decomposition (DMD) method. The effectiveness of this algorithm is represented in a numerical case study of a multi-DOF but one-dimensional task, where the accuracy can also be traced. Then, the DMD method is also tested in real-world industrial, three directional cases where the experimental FRF's of machine tool structures are decomposed and the modal parameters are extracted. Finally, the large-scale modal test of an industrial fan is evaluated by the proposed method.

2. Introduction of impulse dynamic subspace

Henceforward, the experimental FRF's are collected in a matrix function $\mathbf{H}(\omega)$ by using general non-proportionally damped consideration of the modes

$$\mathbf{H}(\omega) = [\mathbf{H}_{i,j}(\omega)] = \sum_{l=1}^{N_\mathrm{m}} \frac{Q_l \mathbf{v}_l \mathbf{w}_l^\mathsf{T}}{\mathrm{i}\,\omega - \lambda_l} + \frac{\overline{Q}_l \overline{\mathbf{v}}_l \mathbf{w}_l^\mathsf{H}}{\mathrm{i}\,\omega - \overline{\lambda}_l}, \tag{1}$$

where N_m modes are considered by their poles $\lambda_l \in \mathbb{C}$, modal scaling factors $Q_l \in \mathbb{C}$, modal participation vectors $\mathbf{w}_l \in \mathbb{C}^{Dn}$ and their mode shapes $\mathbf{v}_l \in \mathbb{C}^{Bm}$. In practice, incomplete FRF is defined as $\mathbf{H}(\omega)$: $\mathbb{R} \to \mathbb{C}^{B\,m \times D\,n}$, where B and D are the number of selected spatial dimensions related to m number of sensing points and n number of excitation points. Let us define the FRF in the way that $\mathbf{H}_{i,j}(\omega) = [H_{i,j;k,l}(\omega)]$, where $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$ and $k, l \in \{x, y, z\}$.

For the sake of simplicity, we assume $D = B$ and $n \le m$, which is theoretically equivalent to its opposite case ($m \le n$). Also, direct