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Broadband liner impedance eduction for multimodal acoustic propagation in the presence of a mean flow

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A new broadband impedance eduction method is introduced to identify the surface impedance of acoustic liners from *in situ* measurements on a test rig. Multimodal acoustic propagation is taken into account in order to reproduce realistic conditions. The present approach is based on the resolution of the linearized 3D Euler equations in the time domain. The broadband impedance time domain boundary condition is prescribed from a multipole impedance model, and is formulated as a differential form well-suited for high-order numerical methods. Numerical values of the model coefficients are determined by minimizing the difference between measured and simulated acoustic quantities, namely the insertion loss and wall pressure fluctuations at a few locations inside the duct. The minimization is performed through a multi-objective optimization thanks to the Non-dominated Sorting Genetic Algorithm-II (NSGA-II). The present eduction method is validated with benchmark data provided by NASA for plane wave propagation, and by synthesized numerical data for multimodal propagation.

1. Introduction

Nacelle manufacturers need to evaluate liner acoustic performance in realistic operating conditions, and inverse techniques are naturally well-suited to identify the impedance properties *in situ*. Only measurements of the sound field properties at selected locations outside the liner region are involved, and furthermore, the liner sample is not destroyed by drilling holes for mounting transducers during these experiments. In order to perform this impedance identification, a numerical model is required for sound propagation in the treated duct, taking account of the presence of a mean flow and of a locally reacting liner. The liner impedance is then estimated by minimizing the error between the calculated and measured sound fields along the duct. Various numerical models for indirect approaches have been proposed, but they have been mostly validated for plane wave propagation. The aim of the present study is to develop an impedance eduction method that can be used for a larger frequency band, in order to consider multimodal acoustic propagation.

The considerable work initiated by Watson et al. [1,2] must be first mentioned. They have developed an impedance eduction method and validated it by comparison with well-documented experiments. A rectangular duct with a ceramic tubular liner section and the presence of a mean flow is considered. The pressure is measured along the duct wall for

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Nomenclature

A_k, B_k, C_k, Z_∞ coefficient of the multipole model (8)
B half-width of the source (2)
c_0 speed of sound (air)
d liner core depth
f frequency
H Heaviside function
$\mathrm{i} = \sqrt{-1}$ unit imaginary number
I sound intensity
IL insertion loss
$k = \omega/c_0$ wavenumber
L_1,L_2 liner's coordinates on the x-axis
L_x, L_y, L_z length, width and height of the duct
$M = V_b/c_0$ Mach number
n_y, n_z velocity profile parameters
p pressure
P number of real poles in Eq. (8)
Q source term in LEE (1)
S number of complex conjugate pole pairs in Eq. (8)
t time
$\mathbf{v} = (v_x, v_y, v_z)$ velocity
$\mathbf{V_0} = (V_{0x}, V_{0y}, V_{0z})$ mean flow velocity
V_b bulk velocity
W transmitted sound power
$W_{\mathrm{ref}} = 10^{-12}$ W reference sound power
$\mathbf{x} = (x, y, z)$ spatial coordinates
$\mathbf{x_S}$ spatial coordinates of the source
Y surface admittance
Z surface impedance

Greek characters

$\alpha_k \pm \mathrm{i}\beta_k$ complex conjugate pole in Eq. (8)
δ Dirac delta function
$\Delta t, \Delta x, \Delta y, \Delta z$ temporal and spatial steps
λ_k real pole in Eq. (8)
ρ_0 density (air)
$\omega = 2\pi f$ angular frequency

Subscripts

L lined duct
R rigid duct

Superscripts

^ Fourier transform, see Eq. (5)
~ admittance model coefficient (11)
* complex conjugate

different Mach numbers up to M=0.4, frequencies from 500 to 3000 Hz and source levels at 120, 130 and 140 dB. A benchmark database has been created and is still used in numerous studies for validation purposes. Their eduction method is based on the resolution of the convected Helmholtz equation using a finite-element method for computing sound propagation, and pressure measurements on the rigid walls. An analytical mode matching approach has also been used to develop identification methods. In the studies by Elnady et al. [3] and Sellen et al. [4], the duct is divided into three zones (unlined–lined–unlined). The acoustic field in each zone is expanded in terms of the duct eigenmodes, taking into account the associated boundary condition on the duct walls, and is solved by a mode matching method. The minimization process relies on pressure measurements upstream and downstream of the lined section [3]. Mode matching methods offer the advantage of taking high-order modes into account with a competitive computational time, but the convected Helmholtz equation is solved for a simple uniform flow. Richter et al. [5] have developed a time-domain eduction method combining the linearized Euler equations (LEE) and a five parameters extended Helmholtz resonator model for the impedance model. Results have been found significantly improved by taking into account the measured velocity profile in the propagation step. Eversman and Gallman [6] have included an effective mean flow Mach number and the exit impedance of the test channel in the objective function of their eduction process. The inverse method is again performed with a finite-element model. Such extended eduction process improves the impedance identification, in particular for high Mach number flows. Piot et al. [7] and Primus et al. [8] have developed an impedance eduction method which relies on the minimization of the squared error between the acoustic velocity provided by the numerical model and experiments. The linearized Euler equations are solved with a discontinuous Galerkin algorithm, and the acoustic velocity field in a plane perpendicular to the liner surface is measured by laser Doppler anemometry.

All these eduction methods provide good results for plane acoustic waves. However, the acoustic liners are designed for applications involving multimodal propagation, and eduction methods must be generalized to test liners mounted in industrial rigs with more realistic conditions. This is of particular interest for non-locally reacting liners, that cannot be characterized by plane wave at grazing incidence. In the recent work by Watson et al. [9] non-planar waves are considered with multiple higher-order modes in the direction perpendicular to the liner and to the opposite rigid wall. Their eduction process is based on a microphone array mounted on a wall adjacent to the liner. Using measured data in a duct for which several higher-order vertical modes can be separated, the authors have shown that each of these modes is submitted to the same local-reacting impedance at the liner surface. More recently, Buot de l'Epine et al. [10] have investigated a rectangular duct where at least two modes can be cut-on. An eduction method formulated from a Bayesian approach is presented to identify the liner impedance.

In this work, an eduction method suitable for multimodal acoustic propagation is proposed. The difference between

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