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Finite element model calibration of a nonlinear perforated plate

David A. Ehrhardt [a,*], Matthew S. Allen [b], Timothy J. Beberniss [c], Simon A. Neild [a]

[a] Mechanical Engineering, University of Bristol, Bristol BS8 1TR, UK
[b] Engineering Physics, University of Wisconsin-Madison, Madison, WI 53706, United States
[c] Aerospace Systems Directorate, Air Force Research Lab, Wright-Patterson AFB, Dayton, OH 45433, United States

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ABSTRACT

This paper presents a case study in which the finite element model for a curved circular plate is calibrated to reproduce both the linear and nonlinear dynamic response measured from two nominally identical samples. The linear dynamic response is described with the linear natural frequencies and mode shapes identified with a roving hammer test. Due to the uncertainty in the stiffness characteristics from the manufactured perforations, the linear natural frequencies are used to update the effective modulus of elasticity of the full order finite element model (FEM). The nonlinear dynamic response is described with nonlinear normal modes (NNMs) measured using force appropriation and high speed 3D digital image correlation (3D-DIC). The measured NNMs are used to update the boundary conditions of the full order FEM through comparison with NNMs calculated from a nonlinear reduced order model (NLROM). This comparison revealed that the nonlinear behavior could not be captured without accounting for the small curvature of the plate from manufacturing as confirmed in literature. So, 3D-DIC was also used to identify the initial static curvature of each plate and the resulting curvature was included in the full order FEM. The updated models are then used to understand how the stress distribution changes at large response amplitudes providing a possible explanation of failures observed during testing.

1. Introduction

Model calibration is an important step in the development of computational models that are representative of physical structures. In this context, there is a large suite of test and analysis approaches which use a structure's linear modes of vibration to guide the calibration of computational models [1,2]. It is beneficial to note here that these techniques can be centered on a structure’s linear modes of vibration using the complex mode definition or a more specific subset of complex modes that are called 'classical modes', 'undamped normal modes', or 'real modes', and here will be simply referred to as 'linear normal modes' (LNMs) of vibration [3]. In many instances, the LNMs, which are dependent only on the mass and stiffness distribution of a structure, are the preferred basis for comparison since damping is often not accounted for in the finite element model (FEM). When LNMs are employed in model calibration, the mass and stiffness can be updated such that they reproduce the measured natural frequencies and (real) mode shapes. However, characteristics of these LNMs such as

* Corresponding author.
E-mail address: david.ehrhardt@bristol.ac.uk (D.A. Ehrhardt).

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amplitude invariance and orthogonality break down when a structure behaves nonlinearly. This has motivated many studies focused on expanding the definition of a structure's LNMs to include nonlinear behavior resulting in several definitions of what are termed Nonlinear Normal Modes.

Two main definitions of nonlinear normal modes (NNMs) can be found in literature [4–7]. The first was developed by Rosenberg [4] for nonlinear conservative (i.e. un-damped) systems and limits a NNM to a *vibration in unison* of the nonlinear system. This definition was later extended to include *non-necessarily synchronous periodic motions* of the nonlinear system, and exploited by Kerschen et al. [6] and Peeters et al. [8] for the numerical computation of NNMs. Although using the conservative nonlinear equations of motion (computed using only mass and stiffness), this extended definition allows the inclusion of internal resonances which can lead to non-synchronous motions of the nonlinear system. It is worth noting this NNM definition has been generalized by Shaw [5], in which an NNM is defined as a two-dimensional invariant manifold in phase space extending the NNM concept to damped systems. The invariant manifold definition of an NNM has also been extended to include internal resonances by defining internally resonant NNMs as a $2m$-dimension invariant manifold where m is the number of modes retained for the definition of the invariant manifold [9–12]. This investigation focuses on the NNM definition used by Rosenberg, Vakakis, and Kerschen, and applies it to the numerical and experimental identification of NNMs of a continuous nonlinear system which is discritized using the underlying linear normal modes.

There have been several applications of undamped NNMs in the field of structural dynamics. For instance, NNMs have been used to provide insight to guide the design of nonlinear vibration absorbers [13] as well as a structure with tunable bending-torsion coupling [14]. NNMs have also been used to characterize FEMs of complicated, geometrically nonlinear structures aiding the creation of accurate nonlinear reduced order models [15,16]. Kurt et al. [17] numerically demonstrated the use of NNM backbone curves to guide the identification of nonlinear stiffness coefficients for a system with local nonlinearities. Of particular interest to this work, NNMs provide a tool to connect computational results with experimental measurements. For instance, NNMs have been used to correlate simulations [18] with experimental measurements [19,20]. NNMs are an excellent tool for model calibration, because they provide a compact means for summarizing the measured or calculated nonlinear behavior of a system in the form of a frequency-energy (or frequency-amplitude) plot and the associated deformation shape. This investigation seeks to extend the use of NNMs to update parameters of a full-order geometrically nonlinear FEM.

The implementation of NNMs for the purpose of model calibration requires advanced techniques in their analytical or numerical calculation as well as their experimental measurement. Analytical techniques include the method of multiple scales [6,7,21,22], normal forms [23], and the harmonic balance approach [24], but are typically restricted to structures where the equations of motion are known in closed form limiting their application to simple geometries and low order systems. Numerical methods have also been developed to calculate a system's NNMs without the approximations required in the analytical approaches [8] and have been used to compute the NNMs of relatively complicated structures [20]. These techniques have been extended to geometrically nonlinear finite element models (FEM) using an approach whereby the NNMs are calculated by coupling numerical continuation to transient dynamic simulation of full order FEM [25]. While numerical techniques are powerful, they are time consuming to implement for a large order FEM making application to iterative procedures (i.e. model calibration) difficult. Therefore in this investigation, the full order FEM is created and updated in Abaqus® and is used to create a nonlinear reduced order model (NLROM) following procedures discussed in [26]. The low order NLROMs are then used to examine NNMs with the use of the continuation routine presented by Peeters et al. [8].

Nonlinear normal modes can also be measured experimentally, but far less has been published in this area due to the difficulty of accounting for damping in the dynamic response of a structure, which is needed to isolate a NNM. Recent work has sought to identify NNMs using phase separation techniques relieving the need to cancel damping in the measurement of a NNM [27]. Alternatively using a phase resonance approach, NNM backbones have been identified from the damped dynamics of a structure using the free decay of a response initiated near a NNM solution [19,28] or the stepped forced response using a multi-frequency input force [11]. The free decay results presented in [19,28] have shown good agreement between calculated and experimentally identified NNMs; however, modal [11] and shaker-structure interactions [29] demonstrate that there is no guarantee a lightly damped transient will follow a NNM. Alternatively, the force appropriation technique used to initiate a free decay in [19] has been extended to identify a NNM by incrementally increasing the input force amplitude and tracking the phase lag criterion along a NNM backbone [11]. This stepped-force technique allows the implementation of multi-frequency inputs to account for damping changes with response amplitude and is used in this investigation.

The goal of this work is to propose and implement a model updating framework that can be used to accurately capture the linear and nonlinear dynamic response of geometric nonlinear structures. The first step in updating a model is to decide which dynamic properties to measure and how to compare them between the model and experiment. In this work we focus on the physical parameters that have potential uncertainty and their effects on the global dynamics of the structure (i.e. LNMs and NNMs). Updates to the initial geometry, material properties, and boundary conditions are considered. Early measurements showed that a small difference between the FEM and actual geometry could change the nonlinear response considerably as discussed in literature [30,31], so the initial shape of the plate was measured to sub-millimeter accuracy using static 3D digital image correlation (3D-DIC). The difference between the calculated and measured LNMs and NNMs were then used to update material properties and boundary conditions since LNMs and NNMs are closely tied to the physics of the real structure. The resulting models are shown to better represent the structure's linear and nonlinear dynamics. It is