Journal of Sound and Vibration 392 (2017) 295–306

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Analysis and design of nonlinear resonances via singularity theory

G.I. Cirillo [a,*], G. Habib [b], G. Kerschen [b], R. Sepulchre [a]

[a] Control Group, Department of Engineering, University of Cambridge, Cambridge, UK
[b] Space Structures and Systems Lab (S3L), Department of Aerospace and Mechanical Engineering, University of Liège, Liège, Belgium

ARTICLE INFO

Article history:
Received 13 June 2016
Received in revised form
2 December 2016
Accepted 29 December 2016
Handling Editor: A.V. Metrikine
Available online 6 January 2017

Keywords:
Nonlinear frequency response
Detached resonance curve
Singularity theory

ABSTRACT

Bifurcation theory and continuation methods are well-established tools for the analysis of nonlinear mechanical systems subject to periodic forcing. We illustrate the added value and the complementary information provided by singularity theory with one distinguished parameter. While tracking bifurcations reveals the qualitative changes in the behaviour, tracking singularities reveals how structural changes are themselves organised in parameter space. The complementarity of that information is demonstrated in the analysis of detached resonance curves in a two-degree-of-freedom system.

1. Introduction

Nonlinear resonances are a matter of increasing concern in engineering structures, and bifurcation analysis is by now a standard tool in the study of responses of systems subject to periodic forcing. While this type of approach has proven effective, its main limitation lies in that the frequency response is not considered as a whole. Rather, bifurcations are studied for fixed values of the forcing frequency. On the contrary, singularity theory with a distinguished parameter [1] (singularity theory in the following), identifies changes in a one-parameter bifurcation diagram, for example in a frequency response. This viewpoint provides a useful complement to the classical use of bifurcation analysis in engineering applications, while employing the same numerical methods [2, Ch.7].

We adopt the framework developed in [3], probably the most successful attempt to use methods of singularity theory (in the broader sense of [4]) in the context of bifurcation problems. The use of this theory is classical in the analysis of the Duffing oscillator [5]. However, to the best of the authors' knowledge, the engineering literature has not pursued the use of such methods in problems of forced oscillations. This is our motivation to highlight the role of singularity theory in engineering applications concerned with forced oscillations.

A concrete application in this paper is the analysis of isolated branches of solutions in the response of a system. Detached resonance curves have been observed for different systems subject to periodic forcing [6–9], as well as in other applications involving nonlinearities [10,11]. Their analysis is not straightforward in the classical framework of bifurcation analysis. In contrast, we argue that singularity theory provides a complete answer to the problem by revealing the organising role of a particular singularity, the asymmetric cusp. We illustrate this result on a two-degree-of-freedom system with cubic springs.

* Corresponding author.
E-mail addresses: gic27@cam.ac.be (G.I. Cirillo), giuseppe.habib@ulg.ac.be (G. Habib), g.kerschen@ulg.ac.be (G. Kerschen), r.sepulchre@eng.cam.ac.uk (R. Sepulchre).

http://dx.doi.org/10.1016/j.jsv.2016.12.044
0022-460X/

The paper starts with the study of a two-degree-of-freedom system in Section 2, which illustrates and motivates the proposed analysis. Section 3 presents those results of singularity theory that have a direct impact on the analysis of nonlinear frequency responses. Section 4 introduces the role of the asymmetric cusp in organising DRCs in parameter space. Section 5 builds on the previous analysis to investigate the inclusion of a fifth-order spring to avoid DRCs. Conclusions are presented in Section 6 (Fig. 1)).

2. A motivating example

To motivate the developments proposed in this paper, a harmonically-forced two-degree-of-freedom system possessing two cubic nonlinearities is considered herein:

$$\begin{aligned} &M\ddot{y} + C\dot{y} + Ky + f_{nl}(y) = gf\cos(\omega t), \\ &M = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}, \quad K = \begin{pmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{pmatrix}, \quad C = \begin{pmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{pmatrix}, \\ &f_{nl}(y) = \begin{pmatrix} \alpha_1 y_1^3 + \alpha_2 (y_1 - y_2)^3 \\ \alpha_2 (y_2 - y_1)^3 \end{pmatrix}, \quad g = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \end{aligned} \tag{1}$$

where the parameters are listed in Table 1. This system represents a nonlinear primary system to which a nonlinear absorber, termed the nonlinear tuned vibration absorber (NLTVA), is attached. Similarly to what is achieved with the classical linear vibration absorber, the NLTVA can maintain two equal peaks in the frequency response of the primary system, and this despite the frequency-amplitude dependence of nonlinear oscillations ([12], see also the insets in Fig. 2). However, one important challenge is that nonlinear systems can exhibit complex and rich dynamics. For instance, Fig. 2d depicts a detached resonance curve (DRC) that is clearly detrimental to the performance of the nonlinear device.

This adverse dynamics was thoroughly investigated using the numerical continuation of periodic solutions and their bifurcations in [13]. Here we complement this analysis with the viewpoint of singularity theory. In the same spirit of what was proposed in [14,15] for forced Hopf bifurcations, we apply singularity theory to a bifurcation diagram identified to the frequency response of the system. The starting point is to reduce the frequency response of (1) to a scalar equation; to obtain it we use the harmonic balance method retaining one harmonic. The results obtained with this approximation are then verified through numerical computation of frequency responses using an algorithm similar to the one used in [16] for the computation of nonlinear normal modes. Appendix A details the reduction of the harmonic balance equations to a scalar equation

$$g(x, \omega, f, k_2, \alpha_2, c_2) = 0, \tag{2}$$

where $x = y_1 - y_2$ is the relative displacement, called the state variable in the terminology of [1]. The frequency ω is the bifurcation (distinguished) parameter.

The outcome of applying singularity theory to (2) is summarised in Fig. 2. At low forcing amplitudes the response resembles that of a linear system (Fig. 2a), but as f is increased parts of it become bistable (Fig. 2b and c, note that part of

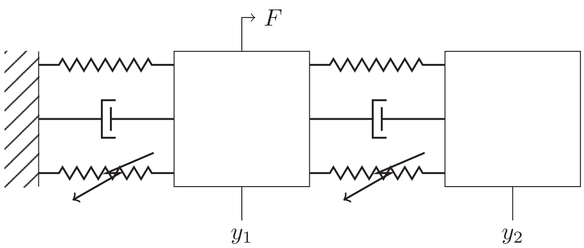

Fig. 1. Nonlinear tuned vibration absorber attached to a primary system.

Table 1
Parameters primary system and absorber.

	primary system	absorber
mass	$m_1 = 1$	$m_2 = .05$
linear stiffness	$k_1 = 1$	$k_2 = 0.0454$
linear damping	$c_1 = 0.002$	$c_2 = 0.0128$
nonlinear stiffness	$\alpha_1 = 1$	$\alpha_2 = 0.0042$