Journal of Sound and Vibration ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Instability phenomena in impact damper system: From quasi-periodic motion to period-three motion

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A R T I C L E I N F O

Article history:
Received 18 May 2016
Received in revised form
9 October 2016
Accepted 5 December 2016

Keywords:
Impact damper system
Quasi-periodic attractor
Jump phenomena
Saddle-node bifurcation
Basins of attraction

A B S T R A C T

The instability phenomena of quasi-periodic attractor in the impact-damper system are reported in this paper. This special phenomena are found by accident when the Hopf bifurcation of impact periodic motion is studied in parameter plane. Quasi-periodic attractor is found to lose stability by sudden jump to period-three attractor or saddle-node bifurcation of period-three attractor on the invariant set. The MDCM (multi-DOF cell mapping) method is used to study the variety of attraction basins of solutions near the critical points of the jump phenomena. Spiral and fragmented attraction basins of solutions in a chosen two-dimensional subspace can be observed.

1. Introduction

The vibro-impact phenomena exist widely in vibration engineering field and the qualitative analysis of periodically-forced vibro-impact oscillators has become one of the most active branch of nonlinear dynamics. The smooth and non-smooth bifurcations observed in the vibro-impact oscillators have been widely studied. For smooth bifurcations, Hopf bifurcation [1–7] plays an important part and large progresses have been made in recent years such as researches on the criteria of Hopf bifurcation [8,9], Hopf bifurcation in the resonance points [10], Hopf interaction with period doubling bifurcation [11], anti-controlling of Hopf bifurcation [12,13]. Meanwhile, the non-smooth phenomena such as grazing, chattering and sliding were also investigated theoretically or numerically. The concept of local discontinuous mapping, firstly introduced by Nordmark [14], laid the foundation for many subsequent studies, such as the sufficient conditions for the persistence of a local attractor in the vicinity of a grazing periodic orbit [15], stability analysis of complete chattering in impacting systems [16], and high co-dimension discontinuity-induced bifurcations [17,18]. Another useful technique called discontinuity geometry [19,20] which was based on the singularity theory to classify the local geometry of the discontinuity set together with associated local dynamics, has been extended to analyze the relationship of saddle-node and grazing bifurcations [21].

Besides the local property, the subsequent evolutionary processes after certain bifurcations are also research hotspots. However, the mechanisms of complex evolutionary processes of attractors [22–26] of impacting system are very complicated and still lack of effective theoretical method. It is well known that secondary bifurcations of the quasi-periodic

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http://dx.doi.org/10.1016/j.jsvi.2016.12.010
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invariant sets can lead to chaos, involving the instability phenomena of quasi-periodic attractor through torus doubling bifurcation or phase locking. However, to our best knowledge, the instability phenomena that the quasi-periodic attractor can lose its stability by sudden jump to period-three attractor or saddle-node bifurcation of period-three attractor on the invariant set are not reported yet.

The purpose of this paper is to report this kind of special instability phenomenon of quasi-periodic attractor in the impact damper system. These peculiar bifurcation phenomena are found when the Hopf bifurcation of impact periodic motion is under study in the chosen parameter plane. And an interesting phenomenon can be observed that the attraction basins of the quasi-periodic attractor near the critical parameter point of the jump phenomena are spiral and fragmented in the chosen two-dimensional subspace.

The paper is organized as follows. Section 2 introduces the mechanical model of impact damper system. Section 3 consists of three parts. A brief introduction to theory of Hopf bifurcation is given firstly in Section 3.1. Secondly, the instability phenomena of quasi-periodic attractor in the impact damper system are presented in Section 3.2. Then the variety of attraction basins of solutions near the critical parameter point of the jump phenomena is studied by the MDCM (multi-DOF cell mapping) method in Section 3.3. The paper is completed by the conclusions in Section 4.

2. Mechanical model

The schematic model of impact damper system is shown as Fig. 1. The mass M_2 can move freely and mass M_1 is connected to the wall by linear springs with stiffness K and linear viscous damper C. The external forces applied to the mass M_1 is harmonic force with amplitude P. And the friction is not under consideration. When the relative displacement of masses satisfies $|X_2 - X_1| = D$, the impacts occur.

The non-impact motion equations between the two consecutive impacts ($|X_2 - X_1| < D$) are

$$\begin{cases} M_1 X''_1 + CX'_1 + KX_1 = P\sin(\Omega T + \tau) \\ X''_2 = 0 \end{cases} \tag{1}$$

Introducing the following non-dimensional quantities,

$$\mu_m = \frac{M_1}{M_2},\quad t = T\sqrt{\frac{K}{M_1}},\quad x_i = \frac{KX_i}{P},\quad \zeta = \frac{C}{2\sqrt{KM_1}},\quad \delta = \frac{KD}{P},\quad \omega = \Omega\sqrt{\frac{M_1}{K}} \tag{2}$$

the system (1) now can be transformed into the following non-dimensional form:

$$\begin{cases} \ddot{x}_1 + 2\zeta\dot{x}_1 + x_1 = \sin(\omega t + \tau) \\ \ddot{x}_2 = 0 \end{cases} \tag{3}$$

When impacts occur ($|x_2 - x_1| = \delta$), according to the law of momentum conservation, the impact equations can be expressed as follows,

$$\begin{cases} \mu_m\dot{x}_{1+} + \dot{x}_{2+} = \mu_m\dot{x}_{1-} + \dot{x}_{2-} \\ \dot{x}_{2+} - \dot{x}_{1+} = -r(\dot{x}_{2-} - \dot{x}_{1-}) \end{cases} \tag{4}$$

where '+' represents the moment after impact and - represents the moment before impact, and the coefficient 'r' stands for the coefficient of restitution. In order to analyze the dynamical behaviors of the impact damper system, the general solutions of system (3) are expressed as follows

$$\begin{aligned} x_1(t) &= e^{-\zeta t}(C_1\cos(\omega_d t) + C_2\sin(\omega_d t)) + A\sin(\omega t + \tau) + B\cos(\omega t + \tau) \\ x_2(t) &= C_3 t + C_4 \end{aligned} \tag{5}$$

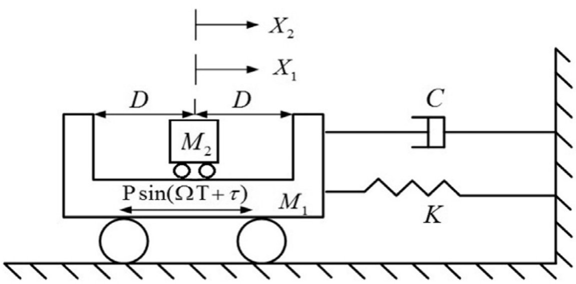

Fig. 1. Schematic model of impact damper system.