Journal of Sound and Vibration ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

A two-dimensional solution of the FW-H equation for rectilinear motion of sources

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ARTICLE INFO

Article history:
Received 22 August 2015
Received in revised form
26 September 2016
Accepted 24 October 2016

Keywords:
FW-H equations
Two-dimensional
Far-field
Frequency domain
Rectilinear subsonic motion

ABSTRACT

In this paper, a subsonic solution of the two-dimensional Ffowcs Williams and Hawkings (FW-H) equation is presented for calculation of noise generated by sources moving with constant velocity in a medium at rest or in a moving medium. The solution is represented in the frequency domain and is valid for observers located far from the noise sources. In order to verify the validity of the derived formula, three test cases are considered, namely a monopole, a dipole, and a quadrupole source in a medium at rest or in motion. The calculated results well coincide with the analytical solutions, validating the applicability of the formula to rectilinear subsonic motion problems.

1. Introduction

Computational aeroacoustics (CAA) has evolved into an essential tool for aerodynamic noise prediction during the past few decades, due to the increasing availability and capability of computational resources. However, achieving high accuracy noise predictions at a reasonable computational cost remains a challenge for realistic applications. Hybrid CAA approaches have been widely used to this end, since decoupling calculation of noise generation from noise propagation, significantly increases computational efficiency of far-field noise predictions.

Application of hybrid CAA is based on an aerodynamic simulation of the noise source region, the results of which are introduced as the inputs to the acoustic solver that may propagate noise in the far-field [1–3]. Acoustic analogy solvers based on the Ffowcs Williams and Hawkings (FW-H) equation [4] are widely used for far-field noise prediction. However, noise source identification remains costly, particularly for three-dimensional geometries and high-Reynolds flows. Therefore, 2D simulations are sometimes preferred, since they can provide an acceptable estimate of noise trends and directivity at a fraction of the computational cost required for 3D simulations.

Flow and noise phenomena are by nature three-dimensional. Thus, 3D computational approaches should be applied if an accurate representation of the flow and acoustic field is required. On the other hand, some flow and noise mechanisms may be considered quasi-two-dimensional (i.e. long span configurations). 2D simulations find increased applicability in such cases, since they can capture noise trends, directivities and general patterns, tending however to overpredict the amplitude.

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http://dx.doi.org/10.1016/j.jsv.2016.10.035
0022-460X/

Nomenclature

c_0	speed of sound
f	data surface
G	Green's function
H	Heaviside function
$M_{\infty i}$	inflow Mach number
$\hat{n}_i$	surface unit normal
p	pressure
p'	acoustic pressure fluctuation
P_{ij}	compressive stress tensor
r	distance between observer and source
R^*	acoustic radii
S	data curve length
$U_{\infty i}$	inflow velocity
t	observer time
T_{ij}	Lighthill stress tensor
u_i	flow velocity
v_i	data surface velocity
v_{oi}	observer velocity
x_i	observer position
y_i	source position

Greek letters

δ	Dirac delta function
δ_{ij}	Kronecker delta
ρ	density
ρ_0	undisturbed flow density
ρ'	acoustic density fluctuation
σ_{ij}	viscous stress tensor
τ	source time

The usefulness of 2D approaches has been shown, among others, by Singer et al. [5] who compared 2D and 3D solutions of slat noise prediction, while Choudhari et al. [6] demonstrated the capability of 2D studies to depict the effect of changes of mean flow velocity and angle of attack. Since many airframe components exhibit a largely two-dimensional acoustic behavior, 2D studies are commonly used to provide estimates of noise generation and propagation at affordable computational costs, even for fairly complex test cases and configurations [7–9].

For three-dimensional studies, time domain formulations are generally preferred [10–12]. Farassat's formulations 1 and 1A [12] are widely applied to aeronautic applications. On the other hand, two-dimensional solutions of the FW-H equation are generally derived in the frequency domain due to the different nature of the Green's function in two and three dimensions. In three dimensions, the Green's function includes a delta function that may be used to evaluate the time integral. However, in two dimensions, the time integration range needed to capture all 2D effects may be quite large and thus the problem is transformed to the frequency domain [13] by using the Fourier integral representation of the two-dimensional Green's function. The resulting two-dimensional solution requires much less computational effort than that required when applying the three-dimensional version to two-dimensional problems, because of the reduction in integration domains by one.

Efficient formulations for 2D noise prediction have been derived by Lockard [13] and Guo [14]. In both formulations the noise sources are interpreted in the same manner as in three dimensions, while the source domains are reduced by one dimension. In particular, Lockard [13] suggested an efficient frequency domain implementation of the FW-H equation for a fixed data curve and a fixed observer in a moving medium, being applicable to both rigid and permeable data curves. Main advantage of Lockard's formula is its robustness and simplicity, thus being easy to implement and computationally efficient. On the other hand, Guo [14] suggested a frequency domain solution for a data curve in motion, which is only applicable to solid data curves located in a medium at rest, whereas the observer is fixed. This formulation, however, is computationally expensive due to the requirement of calculating one integral in space and one in time. In order to propose a more efficient approach, Guo subsequently simplified the initial formula using a far-field approximation for aeroacoustic characteristics [14]. A formula was thus suggested for observers located in the far-field, which is a computationally fast solution.

It should be clarified that Lockard's formula is only applicable to a fixed source in uniform flow and cannot be used when relative velocity exists between source and observer. On the other hand, the formula introduced by Guo is only applicable to a medium at rest. In addition, the approximations Guo used for far-field predictions results in significant error when a uniform flow is present.

In the present work, a subsonic formulation is developed to calculate noise generated by a moving source in a medium at rest or in a moving medium. Moreover, the formula is simplified for far-field calculations using more accurate approximations than those used in [14]. Three analytical test cases are used to verify the validity of the derived formula, namely a monopole, a dipole, and a quadrupole source in a medium at rest or in a moving medium.

2. Governing equation

The FW-H equation as the general form of the Lighthill acoustic analogy [15,16] is an exact rearrangement of the continuity and Navier-Stokes equations used for calculating noise propagated from arbitrarily moving bodies.

Please cite this article as: A. Bozorgi, et al., A two-dimensional solution of the FW-H equation for rectilinear motion of sources, *Journal of Sound and Vibration* (2016), http://dx.doi.org/10.1016/j.jsv.2016.10.035