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Band gap transmission in periodic bistable mechanical systems

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ABSTRACT

We theoretically and numerically investigate the supratransmission phenomenon in discrete, nonlinear systems containing bistable elements. While linear waves cannot propagate within the band gaps of periodic structures, supratransmission allows large-amplitude waves to transmit energy through the band gap. For systems lacking bistability, the threshold amplitude for such energy transmission at a given frequency in the linear band gap is fixed. We show that the topological transitions due to bistability provide an avenue for switching the threshold amplitude between two well-separated values. Moreover, this versatility is achieved while leaving the linear dispersion properties of the system essentially unchanged. Interestingly, the behavior changes when an elastic chain is coupled to bistable resonators (in an extension of the well-studied linear locally resonant metamaterials). Here, we show that a fraction of the injected energy is confined near the boundary due to the resonators, providing a means of regulating the otherwise unrestrained energy flow due to supratransmission. Together, the results illustrate new means of controlling nonlinear wave propagation and energy transport in systems having multistable elements.

1. Introduction

Phononic materials emerge from the periodic arrangement of small-scale building blocks which, through scattering and resonance phenomena, act to control the propagation of acoustic/elastic waves. This ability is demonstrated by the appearance of band gaps in the frequency spectrum. Outside of these gaps, the phononic material is transparent to vibrational waves, which propagate at different speeds dependent upon frequency and direction. Conversely, within the gaps, the material microstructure scatters and/or absorbs the wave energy, prohibiting transmission into the material in all or specific directions. Through careful design of the microstructure (where "microstructure" refers to the small-scale configuration as opposed to the macroscopic scale), the unique dynamics of phononic materials have been exploited at multiple length scales for a myriad of applications in engineering and physics (see, for example, [1,2] and the references therein). These results, based on a linear theory of wave propagation, pertain to waves of sufficiently small amplitude. Where wave amplitudes are large, the influence of nonlinear mechanisms within the material is apparent in the self-modulation of the wave field. Thus, in addition to microstructure design, wave control in phononic materials may feature an amplitude dependency. Moreover, the elicited nonlinear effects may counter the familiar behavior found in the linear regime.

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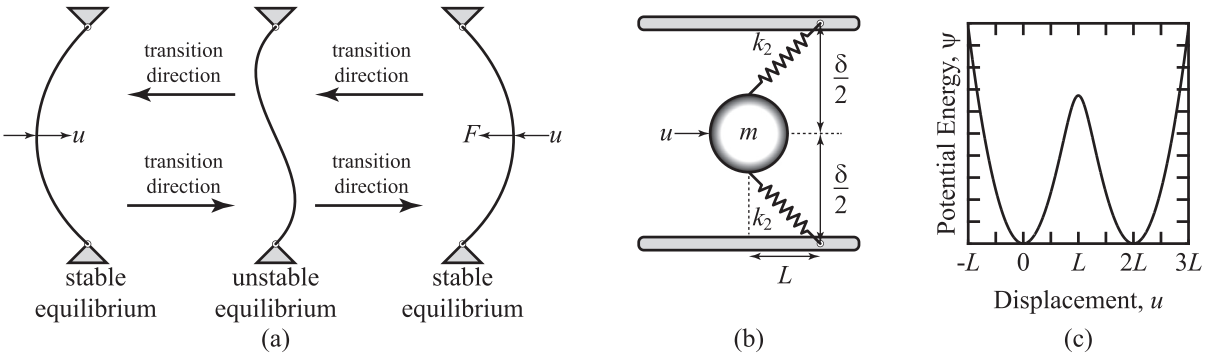

Fig. 1. Mechanical bistability. (a) Buckled column transitioning between three equilibrium states under the influence of a centrally applied lateral load. (b) Mass–spring system as mechanical analog of buckled column. (c) Representative potential energy function $\psi(u)$ shows two local minima (stable equilibria) separated by a local maximum (unstable equilibrium).

Nonlinear supratransmission [3] is the sudden transparency exhibited by discrete nonlinear media subject to continuous boundary driving at a frequency within the linear band gap. Below a critical amplitude, the energy injected into such a system by the driven boundary spatially attenuates away from the driving due to wave reflection and is ultimately removed from the system by the driving. Above this amplitude threshold, the linear evanescent profile is unstable, resulting in the generation of mobile nonlinear modes (breathers, solitons, etc.) [4], i.e., energy transmission within the band gap.

The effect has been observed in a variety of integrable and non-integrable discrete systems, suggesting that it is a generic property of discrete nonlinear networks. While Geniet and Leon [3] introduced the phenomenon in the context of one-dimensional, homogeneous sine-Gordon and Klein–Gordon chains of coupled oscillators, theoretical and numerical work by themselves and others has extended to demonstrations in two-dimensional systems [5,6] and predictions of the threshold amplitude for Bragg [7] and multicomponent media [8]. For non-integrable systems, particularly those externally driven at a frequency outside the continuum limit, the Nonlinear Response Manifold (NLRM) method is a means for determining the critical magnitude of the forcing for supratransmission [9,10]. Utilizing NLRM and simulations, Maniadis et al. [11] predicted multiple forcing magnitude thresholds and Yousefzadeh and Phani [12] showed that, in the practical setting of finite structures, damping may eliminate the supratransmission phenomenon. By inserting an impurity into a discrete, nonlinear chain, Yu et al. [13] were able to control the emission rate of gap solitons and found damping imposed at the impurity improved the profile of the emitted waves. An alternative means of generating the effect which utilizes wave collisions has also been proposed [14]. Nevertheless, in the previous studies, *multistability*, a feature of some nonlinear systems, was either not permitted by the potential energy function or not investigated in detail by the authors. This represents a gap in the literature involving an entire category of nonlinear response. This also forgoes the flexibility of altering the system dynamic performance post-fabrication as demonstrated by Bernard and co-workers for linear waves in a one-dimensional system of locally bistable oscillators [15,16]. As a point of clarity, some works reference nonlinear *bistability* in the context of supratransmission [17,5], however, these articles describe a range of driving amplitudes for which supratransmission and, its counterpart, infratransmission coexist. In this article, rather than describing the coexistence of two dissimilar transmission regions, bistability characterizes a network with two equilibrium configurations.

Multistability (i.e., the possession of two or more stable equilibrium configurations) is a property of a variety of physical [18–21], optical [22], chemical [23–25], and biological systems [26]. In mechanics, a classic example of a bistable system is provided by the buckled column (Fig. 1a). In response to a small lateral load, the column is displaced from its initial equilibrium configuration. With increasing load, the displacement grows until, at a critical value, the system snaps through to a second equilibrium configuration. During the transition between states, such bi- and multistable elements temporarily exhibit negative (static) stiffness, which, when constrained by the environment, can produce composites with effective damping [27,28] and (dynamic) stiffness [29] measures well beyond those of the constituent phases. A chain-like system for energy harvesting from sea waves [30] and recoverable, energy-absorbing cellular media [31] are but a few proposals utilizing bistable elements. In this article, we investigate bistability and the supratransmission phenomenon using a network of mechanically bistable elements previously shown to possess three amplitude-dependent propagation regimes [32].

Among phononic materials, metamaterials [33,34] are distinguished by their extreme/counterintuitive dynamic effective properties [35–39], the macroscale manifestation of subwavelength resonances. Moreover, where conventional phononic materials primarily utilize wave reflection to open spectral gaps, metamaterials exploit an additional wave absorption effect through the use of local resonators. To date, supratransmission has been studied in networks where wave reflection is the band-gap formation mechanism. It remains an open question how supratransmission, which permits energy propagation in the band gap, manifests in systems with an energy absorption capability and what impact resonator bistability (or lack thereof) imparts. Addressing these questions is an additional focus of this article for which we employ a second chain of oscillators with internal, bistable resonators.

The systems considered in much of the supratransmission literature model the behavior of optical waveguide arrays and