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Slow motions in systems with fast modulated excitation

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ABSTRACT

It is well known that high-frequency excitation can modify the behavior of systems with respect to slow motions. The goal of this study is consideration of these effects in a rather general case of analytical systems with modulated sinusoidal excitation. The method of direct separation of motions proposed by I.I. Blekhman was applied in a modified form with the explicit introduction of a small parameter. Equations for the slow motions are obtained and an analysis of how they depend on the structure of the original equations is performed. Five basic effects corresponding to different possible dependencies of the modulation amplitude on position, velocity, and slow time are selected (some of them for the first time). These effects offer a possibility for designing a high-frequency control of the slow motions with specified properties. For example, high-frequency excitation in a system with a nonlinear friction can essentially increase the effective damping. The results are also of significance for system identification and diagnostics. Analysis of a hydraulic valve is given as an example of application.

1. Introduction

1.1. Vibrational mechanics

The concept of vibrational mechanics and the method of direct separation of motions were proposed by I.I. Blekhman [1] and systematically presented in books written by him [2,3]. This approach was further developed and applied to many practical problems by Fidlin [4–6], Malakhova [7], Thomsen [8–10], Sorokin [11], Sperling [12], and others. The main idea consists of the following steps. Let a dynamical system be described as

$$\frac{d^2x}{dt^2} = \Theta\left(x, \frac{dx}{dt}, t, \omega t\right) \tag{1}$$

The system must not be a one-mass mechanical oscillator because Eq. (1) is valid for many physical processes. If that is the case, the parameters are assumed to have been normalized so that the mass is equal to 1 unit. In any case, the function Θ is interpreted as a force and can be presented as a sum of slow and fast components F and Φ

$$\Theta = F\left(x, \frac{dx}{dt}, t\right) + \Phi\left(x, \frac{dx}{dt}, t, \omega t\right) \tag{2}$$

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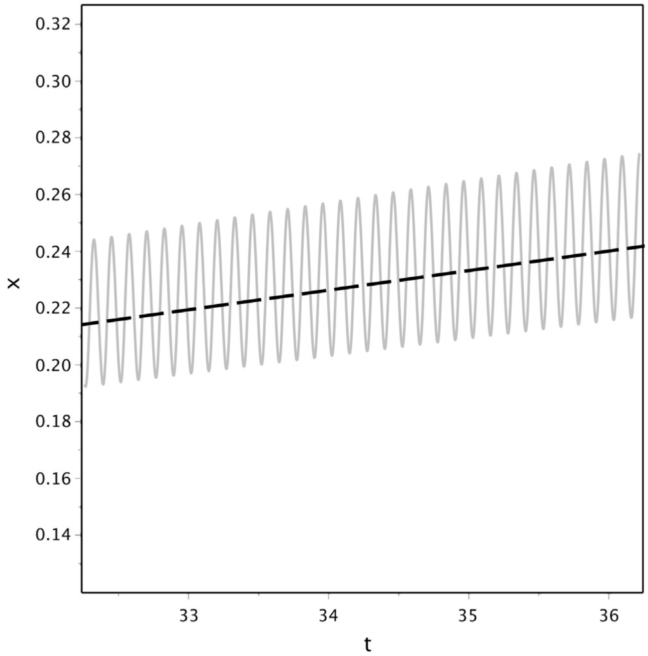

Fig. 1. Character of the solution (solid line – full system, dashed line – averaged system). The units of the axes x and t are here meters and seconds.

A solution can be obtained as a superposition of the time-dependent mean value X and the fast oscillation ψ:

$$x = X(t) + \psi(t, \omega t), \langle \psi \rangle = 0. \tag{3}$$

The function ψ is 2π-periodical and has a mean value equal to 0 with respect to the fast time $\theta = \omega t$. The following is an averaging operation:

$$\langle f \rangle = \frac{1}{2\pi} \int_0^{2\pi} f(t, \theta) d\theta. \tag{4}$$

Fig. 1 illustrates the character of the solution.

The method of direct separation of motions [2] introduces two integral-differential equations for X and ψ as follows:

$$\frac{d^2X}{dt^2} = \left\langle \Theta\left(X + \psi, \frac{dX}{dt} + \frac{d\psi}{dt}, t, \omega t\right) \right\rangle \tag{5}$$

$$\frac{d^2\psi}{dt^2} = \Theta\left(X + \psi, \frac{dX}{dt} + \frac{d\psi}{dt}, t, \omega t\right) - \left\langle \Theta\left(X + \psi, \frac{dX}{dt} + \frac{d\psi}{dt}, t, \omega t\right) \right\rangle \tag{6}$$

Eq. (5) is the result of the direct averaging and Eq. (6) is the complement of Eq. (5) to the original Eq. (1). The aim of further consideration is to obtain an equation for X, which does not include ψ and, therefore, enables the calculation the slow motion without solution of the full system. In order to achieve it, one should obtain ψ through X from Eq. (6) and substitute it into Eq. (5). This leads to an equation for the averaged motion, which is called the *main equation of vibrational mechanics* (also *the equation of strobodynamics* [13] or *the vibration-transformed equation* [2]):

$$\frac{d^2X}{dt^2} = F\left(X, \frac{dX}{dt}, t\right) + V. \tag{7}$$

The additional slow term V in this equation for the averaged motion is the so-called vibrational force, which takes into account the influence of now hidden fast motions. The vibrational force V can depend on the slow variable X, their derivatives with respect to the slow time t, and the slow time t explicit. There are many known nontrivial and unexpected physical effects caused by vibrational forces. Well-known examples are Chelomei's pendulum, the Stephenson–Kapitza pendulum, the Indian rope, vibrational transportation, and many other phenomena presented in the books written by Blekhman [2,3]. Thomsen [8,9] selected three effects, which are characteristics of many different systems with fast excitation – stiffening, biasing, and smoothening. These approaches to systematic effects were developed by him, later together with Fidlin [5] for strong excitation, especially in systems with friction.