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On transversal oscillations of a vertically translating string with small time-harmonic length variations

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ABSTRACT

In this paper, the free transverse vibrations of a vertically moving string with a harmonically time-varying length are studied. The string length variations are assumed to be small. By using the multiple-timescales perturbation method in conjunction with a Fourier series approach, we determine the resonance frequencies and derive the non-secularity conditions in the form of an infinite dimensional system of coupled ordinary differential equations. This system describes the long time behavior of the amplitudes of the oscillations. Then, the eigenvalues of the obtained system are studied by the Galerkin truncation method, and applicability of this method is discussed. Apart from this, the dynamic stability of the solution is investigated by an energy analysis. Additionally, resonance detuning is considered.

1. Introduction

Translating continua are found in many engineering devices as their slender elements such as cables, belts, ropes, tapes, and so on. In mechanics and applied mathematics these elements are usually represented by a string. In this work, we study a vertically translating string with a time-varying length. A rigid body of mass m is attached at the lower end of the string. The string length fluctuates about a constant mean length l_0 as follows $l(t) := l_0 + \beta \sin \Omega t$, where $\beta := \epsilon \beta_0$ is the length variation parameter and Ω is the angular frequency of the length variation. It should be noted that this system can serve as a "toy" model to describe the transverse vibrations of an elevator cable system. A schematic model of this cable system is illustrated in Fig. 1, where a positive or negative transport velocity determines extension (a) or retraction (b) of the cable, respectively. The main objective of this paper is to study the dynamic stability of the cable in case when the frequency of length fluctuation Ω coincides with one of the natural frequencies of the cable, consequently, causing resonance.

In order to restrict the scope of the analysis, it is necessary to impose some assumptions:

- The string is uniform, that is, the linear density ρ is constant.
- Bending stiffness is neglected.
- Only free lateral vibrations, $u(x,t)$, are considered.

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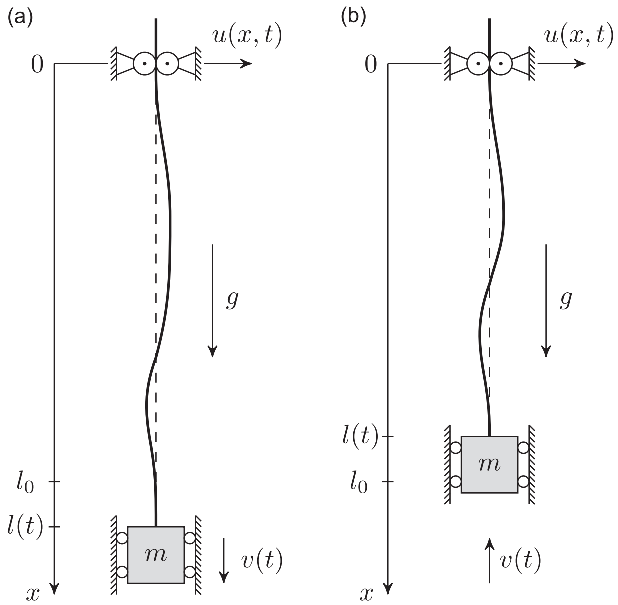

Fig. 1. Schematics of an extended (a) or retracted (b) cable with an attached mass at $x = l(t)$.

- The ends of the string are fixed in the horizontal direction.
- The mass of the string is small compared to the mass of the rigid body, hence providing "constant" pretension in the string. It is worth mentioning that this assumption limits the coefficients of a corresponding eigenvalue problem to be constant, thus simplifying the analysis of the eigenmodes.
- The axial velocity of the string, $v(t) := \dot{l}(t)$, is smaller than the velocity of the wave propagation, where the overdot denotes time differentiation.
- The axial string acceleration, $\dot{v}(t)$, is much smaller than the gravitational acceleration, g.

All in all, the aforementioned assumptions are reasonable assumptions in applications for elevator cables.

Similar problems investigating dynamics of the translating media with varying length or/and velocity have been studied by many researchers. For example, Yamamoto et al. [1] provided a theoretical analysis of free and forced vibrations of a string with time varying length by a power series expansion of the solution. The solution showed agreement with the experimental analysis for the forced vibration. Cooper [2] studied the behavior of the solution for the vibrating string with a moving boundary. Zhu and Ni [3] investigated energetics and stability of translating beam and string models with an arbitrarily varying length. Then Zhu et al. [4] presented the general control laws for the same type of problems in order to provide dissipation of the vibratory energies of translating beams and strings. Some works have been performed particularly for the investigation of the elevator cable dynamics and control (see, e.g., [5–8]). Kaczmarczyk and Ostachowicz derived a mathematical model [9] and presented a simulation [10] of the transient vibration phenomena in deep mine hoisting cables. Chen and Yang [11] investigated the stability of axially accelerating viscoelastic beams by the method of multiple scales. Sandilo and van Horssen [12] constructed formal asymptotic approximations of the vibrations of a vertical string with changing linearly in time length. The same authors also studied autoresonance phenomena [13] in a similar type of problem with harmonic excitation at the top end. The interior layer analysis provided three timescales for the construction of the asymptotic approximations of the transversal vibrations. It should be noted that the current work is a continuation of [12] and of the paper by Gaiko and van Horssen [14], where only the first resonance frequency was analyzed.

The rest of the paper is organized as follows. Section 2 introduces the equation of free lateral vibrations of the cable with a harmonically varying length. In Section 3, the two-timescales perturbation method combined with a Fourier series expansion of the solution is used. This approach allows us to determine resonance frequencies and to obtain non-secularity conditions for the amplitudes of vibrations. Moreover, detuning of the resonance frequencies is also considered. Section 4 proceeds with the investigation of the infinite dimensional system for the amplitudes of the vibrations. Further, in Section 5, we derive the rate of change of the energy and draw some conclusions for resonances and their detuning. Finally, Section 6 summarizes all the results and provides some discussion on the solvability of the current problem.