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Non-probabilistic stability reliability measure for active vibration control system with interval parameters

Yunlong Li, Xiaojun Wang*, Lei Wang, Weichao Fan, Zhiping Qiu

Institute of Solid Mechanics, Beihang University, Beijing 100191, China

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ABSTRACT

A systematic non-probabilistic reliability analysis procedure for structural vibration active control system with unknown-but-bounded parameters is proposed. The state-space representation of active vibration control system with uncertain parameters is presented. Compared with the robust control theory, which is always over-conservative, the reliability-based analysis method is more suitable to deal with uncertain problem. Stability is the core of the closed-loop feedback control system design, so stability criterion is adopted to act as the limited state function for reliability analysis. The uncertain parameters without enough samples are modeled as interval variables. Interval perturbation method is employed to estimate the interval bounds of eigenvalues, which can be used to characterize the stability of the closed-loop active control system. Formulation of defining the reliability of active control system based on stability is discussed. A novel non-probabilistic reliability measurement index is discussed and used to determine the probability of the stability based on the area ratio. The feasibility and efficiency of the proposed method are demonstrated by two numerical examples.

1. Introduction

Active control of vibration is an important concern in the design of aerospace, as the demands for reduction of structural vibrations are increasing and conventional passive vibration control is difficult to meet the design requirements [1,2]. For high frequency vibration, passive control is effective and efficient. However, for the low frequency vibration, passive control is inefficiency. Meanwhile, conventional passive vibration control need vibration isolation material, which is bulky and will significantly increase the weight of the aircraft [3]. As the design of structure must meet the precise and tolerant of vibration, active vibration control is becoming increasingly important and meaningful. The active control system is a combination of sensors to measure the responses of the structure, actuators to apply secondary forces onto the structure and control law to determine how the control force is applied. That is, the closed-loop active control system refers to a complex multi-disciplinary problem, including material mechanic, structural dynamics, signal processing and control theory [4]. For complex system, uncertainties are inherent such as material properties, geometric error and measurement error [5]. It is well known that uncertainties in the active control system can degrade the control performance and even lead to system instabilities. Thus, it is essential to explore the influence of uncertainties in the active vibration control system.

Traditional techniques dealing with the uncertainties in the field of control theory is using the worse-case measure of the

* Corresponding author.
E-mail address: xjwang@buaa.edu.cn (X. Wang).

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system, which is called robust analysis and robust design [6]. Zames firstly proposed the H_∞ control theory in 1981 [7]. Then, the robust control theory was widely studied, and the active control methods were proposed to suppress the structural vibration in 1980s [8–11]. Some representative research has emerged since then. Kar et al. proposed a static state feedback controller to control the bending and torsional vibration of a flexible plate by using H_∞-based control theory [12]. Li and Yam proposed a model-based fuzzy controller to achieve the robust vibration suppress by updating the coefficient matrix online [13]. Hu and Ng used integrated piezoelectric actuators to control the vibration of the flexible structure, the combination of standard LQR (linear quadratic regulator) and nonlinear control signal are adopted to deal with the uncertainties in the closed-loop control system [14]. A robust control strategy was proposed by Zuo and Slotine based on frequency-shaped sliding control theory [15]. For some actual problems of active vibration control of engineering structures, the closed-loop control system obtained based on robust control theory is always over-conservative [16]. That is, robust control theory can achieve the requirement, but the active control system may consume more energy or need greater active control force (secondary force). Generally speaking, robustness is a much stricter constraint than reliability in the field of active vibration control. In order to reduce the conservation, reliability-based control theory is more suitable for dealing with the uncertainties in the active vibration control system [16,17]. To design a reliable controller, reliability analysis of the active control system is essential. Spencer et al. employed first and second order reliability methods (FORM/SORM) to calculate the probabilistic reliability measures of a controlled structure [18], and the stability of the closed-loop control system was chosen as the state limited function [19]. Breitung et al. proposed a reliability analysis method for nonlinear control system, and similarly FORM/SORM methods based on system stability are employed [20]. Taflanidis et al. designed a reliable controller to minimize the probability of structural failure [21]. The most work was done in the probabilistic frame, and generally the probability density function (PDF) requires more information of the uncertain parameters. Interval method as an effective means of dealing with uncertainty can overcome the shortcoming of probabilistic method. Guo employed the infinity norm of the uncertain variables vector to define the reliability, and based on this, a robust reliability method was proposed as a measure of stability of controlled structure [22]. This definition of the reliability proposed by Guo only can measure the case that the shortest distance from the origin to the failure surface is greater than 1. In view of this, Wang et al. firstly proposed the non-probabilistic reliability based on the ratio of the volume of the safe region to the total volume [23]. The definition of non-probabilistic reliability proposed by Wang can measure the case that the reliability is smaller than 1. Then, the non-probabilistic reliability concept was introduced into the reliability analysis of wing flutter [24]. A 15-degree sweptback wing numerical example illustrate the validity of the non-probabilistic reliability index. Non-probabilistic reliability has received considerable attention in the structural design, but in the field of the active vibration control have not been given sufficient attention. Certainly, the final goal is to design a reliable controller. However, the premise of design is the reliability analysis. Thus, only the stability reliability analysis of the active control system is implemented in this paper.

In this paper, an approach for non-probabilistic reliability analysis of active control system is proposed by combining interval perturbation method and non-probabilistic reliability measurement index. The composition of the paper is as follows. In Section 2, there will be a brief review of vibration problem with interval uncertainties. In order to estimate the interval bounds of eigenvalues of closed-loop systems with interval parameters, the interval perturbation method is presented in Section 3. In Section 4 reliability measurement for active vibration control system is developed based on non-probabilistic reliability theory. Numerical examples are provided in Section 5 to show the accuracy and efficiency of the proposed reliability analysis method, followed by conclusions in Section 6. The matrix notation is used as much as possible to simplify the presentation.

2. Problem statement

Consider the following controlled structural system with n degrees of freedom modeled by finite element method

$$\mathbf{M}\ddot{\mathbf{w}}(t) + \mathbf{P}\dot{\mathbf{w}}(t) + \mathbf{K}\mathbf{w}(t) = \mathbf{f}_f(t) + \mathbf{B}_c\mathbf{f}_c(t) \tag{1}$$

where $\mathbf{M} \in \mathbb{R}^{n\times n}$, $\mathbf{P} \in \mathbb{R}^{n\times n}$ and $\mathbf{K} \in \mathbb{R}^{n\times n}$ are the mass, damping and stiffness matrices, respectively. $\mathbf{B}_c \in \mathbb{R}^{n\times s}$ is the location matrix of the control input, $\mathbf{f} \in \mathbb{R}^{s\times 1}$ is control input and s is the number of actuators. The vectors $\mathbf{w} \in \mathbb{R}^{n\times 1}$, $\dot{\mathbf{w}} \in \mathbb{R}^{n\times 1}$, $\ddot{\mathbf{w}} \in \mathbb{R}^{n\times 1}$ are the displacement, velocity, and acceleration vectors, respectively. Here, the vector $\mathbf{f}_f \in \mathbb{R}^{s\times 1}$ represents external disturbance forces, which can be taken to zero in this paper (the external disturbance cannot affect the stability of the system). Ignoring the external disturbance vector $\mathbf{f}_f$, one can transform the vibration Eq. (1) into a state-space equation by using the state vector $\mathbf{x}(t) = \left[\mathbf{w}^{\mathrm{T}}(t), \dot{\mathbf{w}}^T(t)\right]^T$ as follows

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \\ \mathbf{y} &= \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u} \end{aligned} \tag{2}$$

where