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Investigating the sources of variability in the dynamic response of built-up structures through a linear analytical model

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ABSTRACT

It is well established that the dynamic response of a number of nominally identical built-up structures are often different and the variability increases with increasing complexity of the structure. Furthermore, the effects of the different parameters, for example the variation in joint locations or the range of the Young's modulus, on the dynamic response of the system are not the same. In this paper, the effects of different material and geometric parameters on the variability of a vibration transfer function are compared using an analytical model of a simple linear built-up structure that consist of two plates connected by a single mount. Similar results can be obtained if multiple mounts are used. The scope of this paper is limited to a low and medium frequency range where usually deterministic models are used for vibrational analysis.

The effect of the mount position and also the global variation in the properties of the plate, such as modulus of elasticity or thickness, is higher on the variability of vibration transfer function than the effect of the mount properties. It is shown that the vibration transfer function between the plates is independent of the mount property if a stiff enough mount with a small mass is implemented. For a soft mount, there is a direct relationship between the mount impedance and the variation in the vibration transfer function. Furthermore, there are a range of mount stiffnesses between these two extreme cases at which the vibration transfer function is more sensitive to changes in the stiffness of the mount than when compared to a soft mount. It is found that the effect of variation in the mount damping and the mount mass on the variability is negligible. Similarly, the effect of the plate damping on the variability is not significant.

1. Introduction

Uncertainty and variability are inevitable in structural dynamics. There is always a level of variability in manufacturing as well as unavoidable uncertainty in defining the parameters of built-up structures. Increasing the complexity of a built-up structure will result in a higher level of variability in the dynamic response of those nominally identical structures.

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Automotive vehicles are an example of such complex structures with many different parts that possess a high level of variability in their noise and vibration Frequency Response Functions (FRF) (e.g. Refs. [1–5]).

Different methods are adopted to model the uncertainty and variability in structural dynamics [6–10]. Amongst different components, it is often suggested that bolted joints and fasteners are the most variable and uncertain as their properties vary for each individual joint and also over time. A comprehensive review on this uncertainty and the techniques used to model joint variability can be found in Ref. [11]. There have been also attempts to identify and model the variability in other components, for example, Scigliano et al. [12] assessed the variability of a car windscreen due to temperature variation numerically and verified it by experimental results, Resh [13] evaluated the variability in the dynamic characteristics of engine mounts and Donders et al. [14] used a Monte Carlo simulation to assess the effect of spot weld failure on the fundamental natural frequency of a vehicle body-in-white. Gärdhagen and Plunt [15] claimed that the variation in Eigen frequencies and large variation in modal damping ratio in a plate and an acoustic cavity can cause similar variations in FRF to that seen in vehicles. To have such similarity between experimental results and their simple model they used a high variation in modal damping ratio in a way that 70% of modes have a damping ratio in the range of 0.46–3.2%. This is a high level of variation in damping ratio even though it is the most uncertain parameter that needs to be estimated in structural dynamics [16]. Wood and Joachim [17] studied the variability of twelve nominally identical cars and concluded that the damping in the spot welds and joints is the main source of variability. However, these studies do not present a detailed comparison between the contributions made by different parameters of the system to the overall variability. Although they showed that FRF variability can be due to a variation of one of the system parameter, for example the joint properties, the same level of FRF variability can also be produced by a smaller change in another parameter of the system, for example the stiffness of one of the components.

The range over which one parameter can be expected to vary may be very different to that for another parameter. For example, the stiffness of rubber bushes can vary over a wide range while the variation in modulus of elasticity of a plate is very small. Furthermore, the effect of the variation of each parameter on the dynamic response of the structure may not be similar. This paper addresses the latter issue i.e. the effect of different parameters such as structure thickness, material properties, manufacturing tolerances, etc. on the variability of the dynamic response of a built-up structure. An assembly of two plates connected by a single mount at low and medium frequency consistent with interior automotive structure is considered here, where a simple built-up structure has been used in order to make it possible to obtain an analytical model that allows for the examination of the role of the different parameters in the overall variability. The model is linear and the effect of nonlinearity in mounts or other elements of a built-up structure is not considered in this study. A lumped parameter model for the mount is used as in practice the distributed mass of the mount usually affects the vibration transfer problem only at higher frequencies that are beyond the frequency of interest of this paper. The motivation behind this work is to understand the source of variability in noise and vibration FRFs in an automotive vehicle which are comprised of many structural parts and where some of these parts are connected with small mounting clips and joints, for example the attachment of plastic trim components to door panels via plastic push clips. Furthermore, the results can be implemented in methods that allow propagation of the variability from component level to the built-up structure (e.g. Ref. [18]). The frequency range of interest is limited to the low and medium frequency range. In this frequency range deterministic models are typically used for vibration analysis. The limitations of deterministic models at higher frequencies are discussed in Ref. [19] which provides an introduction to Statistical Energy Analysis.

The mathematical model and the derived approximate equations for various extreme cases are given in Section 2 of the paper. This is followed by a numerical example in Section 3 that has been used in this paper for comparison. In Section 4, the effects of different parameters on the vibration transfer function are examined and a sensitivity analysis is used to compare the effect of different parameters on the variability. Sensitivity analysis is widely used in structural design optimisation and its concept is well established [20–22]. Here, a normalised sensitivity function is used for comparison purposes.

2. Mathematical modelling

The mobility and impedance method can be employed to obtain vibration transfer functions of assemblies from a knowledge of the transfer functions of the individual sub-structures [23,24]. In the present work, the method is applied to two connected plates and mobility FRF is obtained. Such configurations are common in vehicles, for example the door trim is connected to the door by a series of clips. To simplify the analysis, only one mount is used in this study, but the same method can be applied to a more complex system (e.g. the representation cited in Ref. [25]). Since the focus of this study is on the low and medium frequency range, only bending modes of the plates are considered.

The schematic representation of the two connected plates is shown in Fig. 1, where the plates are considered simply supported on all four sides. Plate 1 is excited at point 1 and the response is obtained at point 4 on plate 2. The internal forces and velocities are shown at the mounting position on both plates and at the mount location. The points on the mount are referred to by the same number as their corresponding points on the plate with an additional dash.

In the following sub-sections, analytical formulation for out-of-plane mobility FRFs and impedances of a mount that is modelled as a mass-spring-damper is given. The mobility of the assembly as a function of mobility and impedance of its sub-structures is obtained. This is the model that the rest of the paper is based on. Approximate formulations are also given in the last sub-section here that provides an insight into the physics of the problem.