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Benchmark solution for vibrations from a moving point source in a tunnel embedded in a half-space

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ABSTRACT

A closed-form semi-analytical solution for the vibrations due to a moving point load in a tunnel embedded in a half-space is given in this paper. The tunnel is modelled as an elastic hollow cylinder and the ground surrounding the tunnel as a linear viscoelastic material. The total wave field in the half-space with a cylindrical hole is represented by outgoing cylindrical waves and down-going plane waves. To apply the boundary conditions on the ground surface and at the tunnel–soil interface, the transformation properties between the plane and cylindrical wave functions are employed. The proposed solution can predict the ground vibration from an underground railway tunnel of circular cross-section with a reasonable computational effort and can serve as a benchmark solution for other computational methods. Numerical results for the ground vibrations on the free surface due to a moving constant load and a moving harmonic load applied at the tunnel invert are presented for different load velocities and excitation frequencies. It is found that Rayleigh waves play an important role in the ground vibrations from a shallow tunnel.

1. Introduction

The problem of ground-borne vibrations caused by underground moving trains has received considerable attention in the past decades due to the increasing public sensitivity to environmental impacts. In order to predict these vibrations, various analysis models have been proposed by different methods. Treating the tunnel as a simplified beam embedded in a viscoelastic half-plane, simple analytical solutions were proposed to investigate ground vibrations generated by moving loads applied on the beam [1,2]. Balendra et al. [3] developed a plain-strain analytical model to consider the interaction of the tunnel–soil system due to passing trains using a substructure method. By assuming that the diameter of the tunnel was much smaller than the wavelength, Krylov [4] treated each sleeper in an underground track as a point load buried in the half-space for analytical modelling of vibrations from underground railway systems. Later, on the basis of cylindrical elastic theory, a Pipe-in-Pipe analytical model consisting of a circular tunnel wall and a full-space with a cavity was proposed by Forrest and Hunt [5,6] to predict ground vibrations induced by a deep underground railway tunnel. Hussein et al. [7] presented an extension of the Pipe-in-Pipe model for calculating vibrations from underground railways that allows for the incorporation of a multi-layered half-space geometry. In this model, it is assumed that the tunnel displacement is not influenced by the presence of the free surface or ground layers, which is computationally efficient and predicts the overall

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response of the multi-layered half-space well.

The analytical methods mentioned above have the advantage of high computational efficiency and satisfying the radiation conditions naturally. But their application is limited to simple geometries and deep tunnels. The finite element method (FEM) and other discretization methods offer possibilities to account for more practical situations. Gardien and Stuit [8] presented a modular FEM model to quantify the influence of important input parameters for modelling of soil vibrations from railway tunnels. By the boundary element method (BEM), Stamos and Beskos [9] investigated the dynamic response of long lined tunnels buried in a full- or half-space to harmonic or transient external dynamic forces. Andersen and Jones [10] used a coupled FEM–BEM scheme in both two and three dimensions to analyze the vibration transmission from the tunnel to the ground surface.

Fully three-dimensional (3D) numerical approaches give high computational cost, and thus as an alternative a two-and-a-half dimensional (2.5D) method was developed by assuming that the cross-section of the tunnel–soil system is invariant in the longitudinal direction (tunnel direction). In the 2.5D methodology, the Fourier transform applied along the longitudinal coordinate allows to obtain 3D responses from a two-dimensional mesh. In later years, the 2.5D methodology was successfully applied with both FEM, BEM and coupled FEM–BEM. Using the 2.5D finite/infinite element approach, Yang and Hung [11,12] performed a parametric study on soil vibrations caused by underground moving trains, which took into account the dynamic forces resulting from the irregularities between the wheel and rail. Sheng et al. [13,14] developed a 2.5D coupled FEM–BEM method to model ground vibrations from trains running either on the ground surface or in tunnels. Bian et al. [15] used a 2.5D FEM formulation with viscous artificial boundaries to model wave propagation from underground moving loads. Lopes et al. [16] and Amado-Mendes et al. [17] proposed a similar model based on a 2.5D FEM–PML (perfectly matched layers) or a 2.5D MFS (method of fundamental solution)–FEM approach to solve this problem. By assuming the geometry to be periodic instead of invariant in the tunnel direction, Clouteau et al. [18] and Degrande et al. [19] presented a periodic coupled FEM–BEM formulation using the Floquet transform. Based on two boundary value problems (an elastic half-space and a full-space with a cavity), Müller et al. [20] proposed a 2.5D coupled FEM–ITM (integral transform method) model to study the dynamic interaction between a moving vehicle and a plate elastically mounted in a tunnel. However, even with the invariant or periodical geometric property in the tunnel direction, the required computation effort of the 2.5D methodology is still very high compared with the analytical method [7].

To the best of our knowledge, no closed-form three-dimensional semi-analytical solution exists for the ground vibration from a tunnel embedded in a half-space. Thus, the aim of this paper is to provide such a closed-form semi-analytical solution to predict ground vibrations from a shallow or a deep underground railway tunnel, which combines both accuracy and computational efficiency. An analytical solution can not include all situations that occur in practice, but the greatest value of this study is as benchmark for other computations, particularly with FEM. It also serves as an efficient tool to predict ground vibrations induced by underground railway tunnel in early design phases. In this paper, the tunnel is modelled as an elastic hollow cylinder and the ground surrounding the tunnel as an elastic half-space with a cylindrical hole. The total wave field in the half-space with a cylindrical hole is described by outgoing cylindrical waves and down-going plane waves. The transformations between the plane wave functions and the cylindrical wave functions are used to meet the boundary conditions on the ground surface and at the tunnel–soil interface. As the response to any complicated loading condition can be easily determined by a superposition of point-load cases, numerical results for ground vibrations due to a unit moving point load applied at the tunnel invert are presented for different load velocities and frequencies. Furthermore, the influence of the tunnel depth and the tunnel thickness on the propagation of vibrations is investigated.

2. Governing equations and solutions

The tunnel–soil model considered is schematically shown in Fig. 1. The ground surrounding the tunnel is modelled as a half-space with a cylindrical hole made of a linear elastic, homogeneous and isotropic material. The material parameters are the density ρ and the Lamé constants with hysteretic damping λ and μ. The tunnel is modelled as an elastic hollow cylinder with inner radius a and outer radius b (tunnel thickness $h=b-a$). The axis of the tunnel is placed parallel to the traction-free ground surface at a depth d below the surface. The tunnel wall is assumed to be elastic with the density ρ_1 and the Lamé

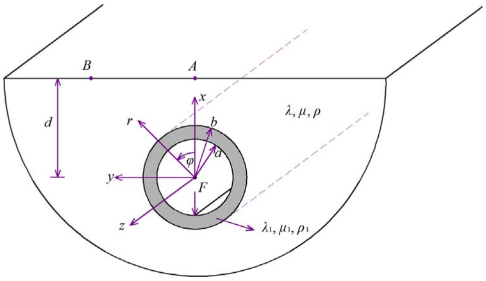

Fig. 1. Geometry of a tunnel embedded in an elastic half-space with a moving point load.

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