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A marginal fractional moments based strategy for points selection in seismic response analysis of nonlinear structures with uncertain parameters

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ABSTRACT

The present paper proposes a new strategy for selecting representative points in the probability density evolution method (PDEM) to conduct stochastic seismic response analysis of nonlinear structures with uncertain parameters. In PDEM, the strategy for selecting representative points in random-variate space is of critical importance to the efficiency and accuracy. The proposed strategy is established based on the marginal fractional moments of input random variables, which can be evaluated both analytically and numerically without difficulty before performing stochastic analysis. In this strategy, an optimization problem is actually involved. First, the initial points are generated by a low discrepancy sequence and the corresponding assigned probabilities can be computed accordingly. Then, the initial points are rearranged to minimize the index, which is adopted as the maximum relative error between the estimated marginal moments and the exact ones. The rearranged points are accepted as the representative points in PDEM when the index reaches the prescribed tolerance. Numerical example is investigated, showing that the proposed strategy can achieve the good tradeoff of efficiency and accuracy in PDEM for seismic response analysis of structures with uncertain parameters.

1. Introduction

For engineering structures, it is almost inevitable to encounter nonlinear behaviors when they are driven by seismic ground motions [1]. On the other hand, randomness has long been observed and widely recognized in both seismic ground motions and structural parameters. Thus, it is of paramount importance to take into account both the nonlinearity and the involved uncertainties to assess the structural performance. In fact, this subject can be categorized as stochastic structural analysis theory and random vibration theory. In the past three decades, extensive developments have been studied. The representative contributions on stochastic structural analysis theory include random perturbation method [2] and orthogonal polynomials expansion method [3–5], however, these methods may be not applicable to general nonlinear structures [6]. In random vibration theory, many useful methods such as the method of moments [7–10], the Fokker-Planck-

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Kolmogorov (FPK) equation method [11], etc. are extensively investigated. Unfortunately, obtaining the probabilistic dynamic response of complex multiple-degree-of-freedom (MDOF) nonlinear structures is still a great challenge. Although Monte Carlo simulation (MCS) [12] and its various improvements [13–15] are versatile regardless of the structure is linear or nonlinear, the computational efforts may still be intractable especially for large-scale nonlinear structures. Thus, Monte Carlo simulation is widely accepted as a checking method for verification of a newly developed method. To summarize, the methods mentioned above may be not able to achieve the tradeoff of accuracy and efficiency in stochastic seismic response analysis of nonlinear structures with uncertain parameters.

Alternatively, the probability density evolution method(PDEM) [6,16,17], which is capable of capturing the instantaneous probability density function, provides a new perspective to tackle such a problem. In PDEM, a governing partial differential equation called generalized density evolution equation (GDEE) is derived according to the principle of preservation of probability [18]. Numerical algorithms for GDEE, especially the selection of representative points in random-variate space, play an important role in keeping the balance between accuracy and efficiency in practical applications [1]. Several endeavors have been made for the selection of points in PDEM such as the dimension reduction method via mapping [19], the tangent sphere method [20] and the number theoretical method [21]. However, when the dimension is large, the efficiencies of these methods still need to be improved. In addition, the adaptability of cubature points as the representative points in PDEM is discussed in details in Ref. [1], where a criterion is put forward to select the appropriate cubature points and weights for PDEM. Recently, Chen and his co-authors propose a generalized-F (GF) discrepancy [22] and develop a GF discrepancy based strategy for points selection [23] in PDEM. Nevertheless, the investigation of points selection in PDEM is still of great significance, which is closely related to the accuracy and efficiency of stochastic seismic response analysis of nonlinear structures.

In the present paper, a new marginal fractional moments based strategy for points selection in PDEM is proposed to carry out seismic response analysis of nonlinear structures with random parameters. This paper is organized as follows. In Section 2, the fundamentals and the numerical algorithms of PDEM are introduced briefly. Then, the new strategy to select representative points in PDEM is put forward in Section 3, where the marginal fractional moments of input random variables are adopted as the indices. In Section 4, a numerical example is studied to verify the efficacy of the proposed strategy in PDEM. Concluding remarks are included in the final section.

2. Fundamentals of PDEM and its numerical algorithms

In this section, the fundamentals of PDEM and its numerical algorithms are firstly introduced.

2.1. Fundamentals of PDEM

Without loss of generality, consider the equation of motion of an MDOF structural system subjected to seismic excitation as follows

$$\mathbf{M}(\Theta)\ddot{\mathbf{Y}} + \mathbf{C}(\Theta)\dot{\mathbf{Y}} + \mathbf{f}(\Theta, \mathbf{Y}) = -\mathbf{M}\boldsymbol{\Gamma}\ddot{x}_g(\Theta, t) \tag{1}$$

Where $\mathbf{M}$ and $\mathbf{C}$ are the $n \times n$ mass and damping matrices, respectively, $\mathbf{f}$ is the $n \times 1$ linear/nonlinear restoring force vector, $\boldsymbol{\Gamma}$ is the $n \times 1$ loading influence matrix, $\ddot{x}_g$ denotes the seismic ground motion applied to the structure, $\ddot{\mathbf{Y}}$, $\dot{\mathbf{Y}}$ and $\mathbf{Y}$ denote the acceleration, velocity and displacement vectors of the structure, respectively, $\Theta = (\Theta_1, \Theta_2, \ldots, \Theta_s)$ represents s basic independent random variables involved in both structural parameters and external excitation with known probability density function $p_{\boldsymbol{\Theta}}(\boldsymbol{\theta})$.

Structural dynamic problem (1) is usually well-posed, which means the solution to Eq. (1) is existent, unique and dependent on the parameter Θ. Generally, besides the displacement and velocity, some other physical quantities (e.g. the stress, internal forces, etc.) denoted as $\mathbf{Z}(t) = (Z_1, Z_2, \ldots, Z_m)^T$ are of practical interest. Thereby, it is convenient to suppose that

$$\mathbf{Z}(t) = \mathbf{H}(\boldsymbol{\Theta}, t), \dot{\mathbf{Z}}(t) = \mathbf{h}(\boldsymbol{\Theta}, t) \tag{2}$$

where $\mathbf{H}$ and $\mathbf{h}$ are deterministic operators.

Because the randomness involved in the structural dynamic system is completely characterized by Θ, the augmented system $\left(\mathbf{Z}(t), \boldsymbol{\Theta}\right)$ is probability preserved, which leads to [6]

$$\frac{D}{Dt}\int_{\Omega_t \times \Omega_\Theta} p_{\mathbf{Z}\Theta}(\mathbf{z}, \theta, t) d\mathbf{z} d\theta = 0 \tag{3}$$

where $p_{\mathbf{Z}\Theta}(\mathbf{z}, \theta, t)$ is the joint PDF of $\left(\mathbf{Z}(t), \boldsymbol{\Theta}\right)$, $\Omega_t \times \Omega_\Theta$ is the distribution domains of $\mathbf{Z}$ at the time instant t and Θ.

After a series of mathematical manipulations, one can obtain the generalized density evolution equation (GDEE) such that [24,25]