Journal of Sound and Vibration ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Modeling and boundary control of translational and rotational motions of nonlinear slender beams in three-dimensional space

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ARTICLE INFO

Article history:
Received 29 June 2016
Received in revised form
18 October 2016
Accepted 29 October 2016
Handling Editor: L. G. Tham

Keywords:
Slender beams
Large motions
Boundary control
Hilbert space
Evolution systems

ABSTRACT

Equations of motion of extensible and shearable slender beams with large translational and rotational motions under external loads in three-dimensional space are first derived in a vector form. Boundary feedback controllers are then designed to ensure that the beams are practically $\mathcal{K}_\infty$-exponentially stable at the equilibrium. The control design, well-posedness, and stability analysis are based on two Lyapunov-type theorems developed for a class of evolution systems in Hilbert space. Numerical simulations on a slender beam immersed in sea water are included to illustrate the effectiveness of the proposed control design.

1. Introduction

Slender beams such as rods in tethered satellite systems, and marine risers or pipes in off-shore applications are often used in practice. Due to a large length-to-diameter ratio, extensibility and shearbility, these beams exhibit large translational and rotational motions under external loads. Large motions include small motions (often referred to as vibrations) but not vice versa. This is because linear or nonlinear partial differential equations (PDEs) describing small motions of the beam are obtained by linearization or Maclaurin series expansions up to the second order of the original nonlinear PDEs governing (large) motions of the beam (see e.g., [1,2]). Since the original nonlinear PDEs governing motions of beams are complex, most of existing works on boundary control (e.g., [3–26]) have extensively considered only small motions (vibrations). Therefore, the above works only guarantee stability of the beam motions in neighborhood of the equilibrium.

Although large motions can lead to a serious/sudden failure such as loop formation (see e.g., [27] for a numerical study of loop formation in a simplified beam induced by a small twisting moment) and collision among risers in a riser bundle, modeling and boundary control of slender beams with large motions have received much less attention. Models of slender beams with large translational motions were addressed in [28,29]. However, these models were developed for numerical study and did not consider rotational motions and boundary dynamics. In [30], boundary control of inextensible and unshearable risers with large translational motions in three-dimensional (3D) space was considered. In [31], although extensible and unshearable risers in 3D space with large translational motions was addressed, the energy due to extension was precluded from the boundary control design. Recently, boundary control of extensible and unshearable slender beams/risers

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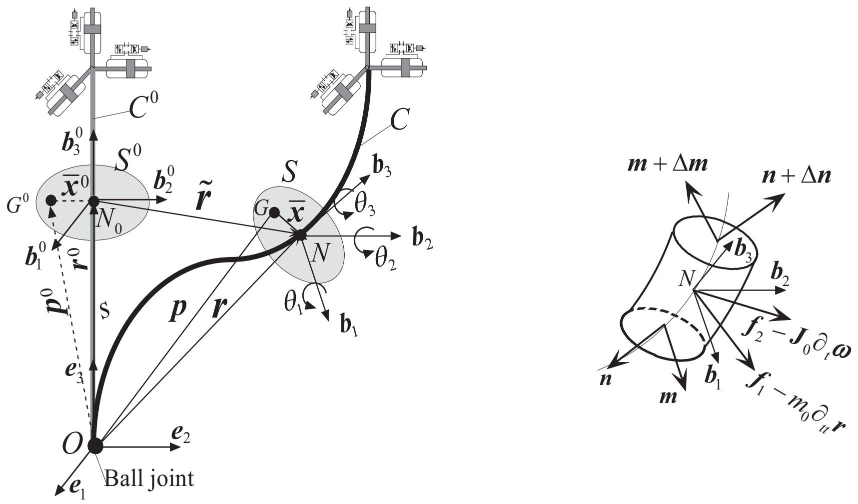
http://dx.doi.org/10.1016/j.jsv.2016.10.044



(a) Deformation geometry of the beam. (b) Forces and moments acting on a beam element.

Fig. 1. Beam deformation geometry and loading diagram on a beam element.

with large motions in two-dimensional (2D) space has been addressed in [32]. In general, slender beams with sufficient stiffness are extensible, shearable, and undergo large motions due to their flexibleness and external loads. Moreover, large motions of slender beams must be avoided as they ultimately kink and break the beams. Thus, from both theoretical and practical points of view, it is necessary to derive PDEs describing large motions of the slender beams and to design boundary controls so as to reduce their motions in 3D space.

The above review motivates the objective of this paper on modeling and boundary control design for extensible and shearable slender beams with large motions in 3D space. The main contributions of this paper consist of two folds.

First, fully nonlinear PDEs in Euclidean space describing large translational and rotational motions of the beams are derived in an appropriate vector form for the boundary control design. In deriving the PDEs, a proper reference configuration and theory of deformation in 3D space are introduced. These PDEs are then transformed to a system of evolution equations described by a set of ordinary differential equations (ODEs) in Hilbert space for convenience of control design and analysis of well-posedness and stability. This above approach has several significant advantages over conventional control design methods dealt directly with PDEs in Euclidean space. Various control design and stability analysis tools such as Lyapunov's direct method [33] developed for lumped-parameter systems in Euclidean space can be mimicked with appropriate functional spaces, norms and inner products being introduced. Moreover, searching for a proper Lyapunov function can be constructive based on the backstepping method [34].

Second, boundary feedback controllers are proposed to practically $\mathcal{K}_\infty$-exponentially stabilize the beam motions at the origin. The control development is based on two Lyapunov-type theorems developed for study of well-posedness and stability for a class of evolution systems in Hilbert space. These Lyapunov-type theorems allow a simple and direct analysis of well-posedness and stability instead of considering each concrete system as in the conventional approach (e.g., [35,6,30,31,36–42]). In the control design, various vector and inner products in both Euclidean and Hilbert spaces, and an appropriate combination of body-fixed and earth-fixed coordinates are also used. The proposed boundary controllers, which are implemented at only one-end of the beam and require only measurements of motions of the beam at this end, guarantee that the beam motions are practically $\mathcal{K}_\infty$-exponentially stable at the origin.

Notation. The symbols $\wedge$ and $\vee$ denote the min and max operators, respectively. These operators are also applied to more than two arguments (e.g., $a \wedge b \wedge c := \min(a, b, c)$, $a \vee b \vee c := \max(a, b, c)$). The symbol "col" denotes the column operator (e.g., $\text{col}(\boldsymbol{a}, \boldsymbol{b}) := [\boldsymbol{a}^T \boldsymbol{b}^T]^T$) while the symbol "row" denotes the row operator (e.g., $\text{row}(\boldsymbol{a}, \boldsymbol{b}) := [\boldsymbol{a}^T \boldsymbol{b}^T]$), where $\boldsymbol{a}$ and $\boldsymbol{b}$ are two column vectors. The symbol "diag" denotes the diagonal operator (e.g., $\boldsymbol{A} = \text{diag}(A_{11}, A_{22}, A_{33})$ means that $\boldsymbol{A}$ is a diagonal matrix with its elements are A_{11}, A_{22}, and A_{33}). The symbol $\times$ denotes the vector cross product operator.

2. Mathematical model

Assume that plane sections of the beam remain plane after deformation, i.e., plane sections are rigid; the beam is locally stiff; and the beam material is homogeneous and isotropic. The beam's deformation geometry is shown in Fig. 1a and the loading diagram on an element is given in Fig. 1b, where the argument t and nonconservative forces and moments are not shown for clarity; and all symbols are defined in what follows.

Please cite this article as: K.D. Do, Modeling and boundary control of translational and rotational motions of nonlinear slender beams in three-dimensional space, *Journal of Sound and Vibration* (2016), http://dx.doi.org/10.1016/j.jsv.2016.10.044