Journal of Sound and Vibration ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Influence of model errors in optimal sensor placement

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ARTICLE INFO

Article history:
Received 5 January 2016
Received in revised form
7 October 2016
Accepted 21 October 2016
Handling Editor: Dr. I. Trendafilova

Keywords:
Optimal sensor placement
Information Entropy
Model error
Model uncertainties

ABSTRACT

The paper investigates the role of model errors and parametric uncertainties in optimal or near optimal sensor placements for structural health monitoring (SHM) and modal testing. The near optimal set of measurement locations is obtained by the Information Entropy theory; the results of placement process considerably depend on the so-called covariance matrix of prediction error as well as on the definition of the correlation function. A constant and an exponential correlation function depending on the distance between sensors are firstly assumed; then a proposal depending on both distance and modal vectors is presented. With reference to a simple case-study, the effect of model uncertainties on results is described and the reliability and the robustness of the proposed correlation function in the case of model errors are tested with reference to 2D and 3D benchmark case studies. A measure of the quality of the obtained sensor configuration is considered through the use of independent assessment criteria. In conclusion, the results obtained by applying the proposed procedure on a real 5-spans steel footbridge are described. The proposed method also allows to better estimate higher modes when the number of sensors is greater than the number of modes of interest. In addition, the results show a smaller variation in the sensor position when uncertainties occur.

1. Introduction

Modern technologies for structural safety, system identification, and damage detection require control systems to monitor the structural behaviour during the whole operating life. These technologies are based on the development of efficient numerical techniques for structural identification and on the adoption of increasingly reliable sensors that conjugate contained costs with performances suitable for the monitoring purposes. However, the quality of the obtained information significantly depends on the number of sensors and on their positions. Usually, several alternative positions can be selected, although economic constraints and spatial restrictions tend to limit the sensor set-up. Thus, it is necessary to optimize the position of a limited number of sensors in order to obtain the maximum amount of information from the measurements and to assure a reliable evaluation of the parameters of interest.

As a matter of facts, sensor placement is a priori problem where only analytical data are available; they are obtained, for instance, by means of a Finite Element Model. Therefore, in a general sensor placement procedure, the estimate of the optimal position is sensitive to errors and uncertainties of the numerical model. They can alter the optimal locations of sensors and limit the efficiency of the monitoring systems. These uncertainties are mainly due to limitations of the adopted numerical models to represent the behaviour of the real structure (*model errors*) and the presence of uncertainties in

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http://dx.doi.org/10.1016/j.jsv.2016.10.033

measures whether experimental data are used (*measurement errors*). Moreover, model errors can be further classified in two groups: parametric uncertainties and model form uncertainties. The parametric uncertainties are associated with the discrepancies between some parameter values of the actual physical system and with the input parameters in the numerical model. Model form uncertainties are associated with the inaccuracy in modeling of the physical system [1].

In this work, the role of parametric and model uncertainties in sensor placements is investigated. The Information Entropy [2] method is applied. According to [3], the covariance matrix of the prediction error is defined by a term counting the measurement error which is assumed as independent from the sensor position and by a term that considers the model error. If the goal of the monitoring process is the modal identification, the Information Entropy only depends on the modal matrix and on the prediction error of the signal acquired by two sensors in hypothetical locations. Thus, results of optimization and placement process considerably depend on the covariance matrix of prediction error and on the definition of the signal correlation. The role of the covariance matrix and the correlation function in optimal sensor placement is thus investigated considering at first a simple case-study without uncertainties. Some approaches in the definition of covariance matrix are compared; firstly it is assumed a constant correlation function and then an exponential correlation function depending on the distance between sensors [3]. A new proposal that depends on both the distance and modal vectors is then presented. The reliability and the robustness of the proposed correlation function in the case of model error are also tested by forcing an alteration (distortion) in a constraint condition. A measure of the quality of the obtained sensor configuration is evaluated through the use of independent assessment criteria. Spatially correlated prediction errors are considered with reference to an asymmetric-plan multi-storey building. Finally, the real case study of a five spans steel footbridge is presented.

The investigation of the model errors (model form uncertainties and parametric uncertainties) in optimal sensor placement and a new proposal for the correlation function are the main contributions of this paper; it will show that the proposed correlation function easily allows to take into account the spatial correlation in 3D structures and it could reduce the variability of results in the case of model uncertainties.

2. Related works

In the last decades, several approaches have been proposed and developed to find a sensor configuration that allows to optimize the number and the position of sensors. An overview of the optimization criteria for optimal placement of sensors and actuators on a smart structure is presented in [4]. One set of criteria was derived from the modal assurance criterion (MAC) originally introduced by [5]. With the aim of leading in uncorrelated mode shapes [6], proposed to minimize the off diagonal terms of the MAC matrix. The MAC or the mass-weighted MAC were recommended as a validation criterion by several authors [7]. In the field of optimal sensor placement [8], proposed to minimize both the average of all the off-diagonal elements and the highest value in all of them.

In [9] a priori analysis was performed; the proposed approach to optimal sensor placement is based on the difference among modal information acquired with or without the contribution of preselected sensors. A lack of information about the *i*-th sensor implies that the mode shape component is estimated by a linear interpolation.

The Effective Independence (EFI) method [10–12] was developed for spatial structures on orbit. It determines the position of each candidate sensor maximizing the determinant of the Fisher Information Matrix (FIM) defined as the product of the mode shape matrix and its transpose. The aim of the EFI method is to select measurement positions that make the mode shapes a readings of interest as linearly independent as possible. The EFI method is extensively discussed in papers [11–14]. The Fisher Information Matrix can also be weighted by the use of the mass matrix carried out by a finite element model [15]. In the EFI method, the number of sensors is iteratively reduced from an originally large candidate set to the desired number, by removing those sensors which do not contribute significantly to the independent information of the mode partition [16]. In the end, the remaining sensors are judged as the optimal sensor set. In [13] it is shown that maximizing the FIM determinant means to maximize the mode shape signal strength and spatial independence.

The determinant of the Fisher Information matrix is often directly considered as the objective function that have to be minimized; other authors suggests to maximize the smallest eigenvalue of the FIM (see for instance [17]), or to minimize the trace of inverse of the FIM [18], or to maximize the norm of the FIM [19,20]. Different variants based on the EFI method were also proposed [21]: for instance, the EFI-DPR (Driving Point Residue) was developed to identify the damage in mechanical elements [10,11,13,22] and the modes are weighted by corresponding driving-point residues.

Several other sensor placement methods are based on energy measures of a structure. These methods usually select locations with high amplitude responses to increase the signal to noise ratio. The Kinetic Energy Method (KEM) [23] maximizes the kinetic energy measure of the structure; however, it is shown that it should be brought back to the maximization of the Fisher information matrix determinant [10]. The idea of maximize the signal to noise ratio of reference sensors is also presented in the method proposed by [24]. Other energy-driven methods are the Eigenvalue Vector Product (EVP) [25] and the Non-Optimal Driving Point (NODP) [26].

Papadimitriou [2] introduces the Information Entropy (IE) as a performance measure of the sensor configuration. The method generalizes several other sensor placement procedures and it demonstrates its efficiency and robustness. Based on the Bayesian approach, the Information Entropy provides for a scalar measure of the uncertainty in the estimate of some model parameters and it depends on the covariance matrix of prediction error. The optimal sensor configuration is the one

Please cite this article as: L. Vincenzi, & L. Simonini, Influence of model errors in optimal sensor placement, *Journal of Sound and Vibration* (2016), http://dx.doi.org/10.1016/j.jsv.2016.10.033