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Structural damage detection based on covariance of covariance matrix with general white noise excitation

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ABSTRACT

Covariance of the auto/cross-covariance matrix based method is studied for the damage identification of a structure with illustrations on its advantages and limitations. The original method is extended for structures under direct white noise excitations. The auto/cross-covariance function of the measured acceleration and its corresponding derivatives are formulated analytically, and the method is modified in two new strategies to enable successful identification with much fewer sensors. Numerical examples are adopted to illustrate the improved method, and the effects of sampling frequency and sampling duration are discussed. Results show that the covariance of covariance calculated from responses of higher order modes of a structure play an important role to the accurate identification of local damage in a structure.

1. Introduction

Damage identification of a structure has received much attention from researchers with structures accumulating damages during their service life. Damage identification can help to determine the physical characteristics and update the finite element model of a structure, which are very important for checking on the capacity of structures for continued service.

The basic idea behind many researches on structural damage identification is to find one indicator which can clearly tell the difference between the damaged and intact structure. In frequency domain, modal frequency which serves as an indicator has been extensively studied in early researches [1–4], while mode shape was introduced later [5–9]. Some studies involved both the mode shape and modal frequency [10]. In the last two decades, the modal strain energy has been widely adopted as an indicator [11–15]. Recently, Mousavi and Gandomi [16] used the modal force error combined with dynamic condensation in the damage detection of a structure.

Another approach to solve this problem is in time domain [17,18]. Wavelet transform is one of the widely used techniques in the time domain approach. Wavelet packet component energy was selected as a damage indicator in some studies [19,20]. Wavelet packet transform was also adopted to analyze the measured response to form the sensitive function in time domain analysis [21]. Law et al. [22] used wavelet coefficient sensitivity for damage detection. Other techniques have also

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been applied to analyze the measured signal. Ren and Sun [23] combined the wavelet transform with Shannon entropy to detect structural damage from measured vibration signals. Liu, et al. [24] applied singular spectrum analysis to enhance the sensitivity in damage detection. An et al. [25] used the jerk energy as the basic indicator to detect damage in a structure. Weng, et al. [26] combined the structural system identification and model updating techniques with verification of the method through a shaking table experiment of a two-bay frame structure.

More recently, Li and Law [27] developed the Covariance of the auto/cross-Correlation (CoC) matrix of accelerations of a structure under ambient white noise excitation as a sensitivity function to detect the local stiffness change or structural damage. They claimed that the CoC matrix is more sensitive to the local damage than mode shape and modal frequency. Law, et al. [28] further investigated the sensitivity of the CoC matrix with respect to a local change in the structure parameter. The expression of the CoC matrix for any selected modes of a structure was derived analytically. Numerical results showed that this method is insensitive to measurement noise when the measurement duration is sufficiently long.

However, previous studies only considered the CoC matrix-based method for structural condition assessment through ambient white noise excitation introduced by support motion, and the analytical CoC matrix with white noise excitations directly acting on a structure was not derived(hereafter in this study we name this kind of excitations as "direct white noise excitations"). Moreover, previous studies showed that the original method requires comparatively large number of sensors for damage detection of a complex structure which will surely limit the application.

In this study, the CoC matrix and its sensitivity to local structure stiffness change is analytically derived with the structure under direct white noise excitations, which is an extension of the original method. The shortcoming of the original damage detection approach is also discussed with improvements proposed.

2. Basic theory

2.1. Acceleration response under direct white noise excitations

The equation of motion of an N-Degrees-Of-Freedoms (DoFs) structural system under white noise excitations is given as:

$$\mathrm{M}\ddot{\mathbf{x}}(t) + C\dot{\mathrm{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathrm{LF}(t), \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are $N \times N$ mass, damping and stiffness matrices of the structure. Rayleigh damping is assumed for the structure. $\mathbf{F}(t)$=[$f_1(t)$, $f_2(t)$,…$f_m(t)$]' is a $m \times 1$ vector representing forces acting at m DoFs of the structure, $\mathbf{L}$ is a $N \times m$ mapping matrix associated with $\mathbf{F}(t)$. The elements of the matrix representing forces can be either null or normally distributed random numbers.

According to the modal superposition theory, the displacement $\mathbf{x}(t)$ can be seen as the summation of all generalized modal displacements $z_i(t)$ (i=1,2,…, N), as

$$\mathrm{x}(t) = \varphi_1 z_1(t) + \varphi_2 z_2(t) + \ldots + \varphi_N z_N(t) = \Phi z(t), \tag{2}$$

where, $\boldsymbol{\varphi}_i$ is the ith mode shape vector, $\boldsymbol{\Phi}$ is the mode shape matrix of the structural system, and$z(t) = \left[z_1(t), z_2(t), \ldots, z_N(t) \right]'$. Applying the orthogonality property, Eq.(1) can be re-written as

$$\Phi' M\Phi\cdot\ddot{z}(t) + \Phi' C\Phi\cdot\dot{z}(t) + \Phi' K\Phi\cdot z(t) = \Phi'\mathrm{LF}(t), \tag{3}$$

where $\boldsymbol{\Phi}$ and $\mathbf{M}$ satisfy $\Phi' M\Phi = \mathrm{I}_N$, $\mathbf{I}_N$ is a $N \times N$ identity matrix. Let $\mathrm{C}_N = \Phi' C\Phi$, $\mathrm{K}_N = \Phi' K\Phi$ which are the generalized $N \times N$ diagonal damping and stiffness matrices. Eq.(1) can then be re-written in the uncoupled form as

$$\mathrm{I}_N\ddot{\mathrm{z}}(t) + \mathrm{C}_N\dot{\mathrm{z}}(t) + \mathrm{K}_N\mathrm{z}(t) = \mathrm{F}_N(t). \tag{4}$$

with $\mathrm{F}_N(t) = \Phi'\mathrm{LF}(t)$. The equation of motion for the ith mode can be written as

$$\ddot{z}_i(t) + c_i\dot{z}_i(t) + k_i z_i(t) = f_i(t) \tag{5}$$

where c_i, k_i are the ith diagonal elements in matrices $\mathbf{K}_N$ and $\mathbf{C}_N$. $f_i(t) = \varphi_i'\mathrm{LF}(t)$ is the ith component of the generalized force matrix. The impulse response function of the ith mode may be written as

$$h_i(t) = \begin{cases} \dfrac{1}{\omega_{Di}}\sin(\omega_{Di}t)\exp(-\xi_i\omega_i t) & t \ge 0 \\ 0 & t < 0 \end{cases}, \tag{6}$$

whereω_i and$\omega_{Di} = \omega_i\sqrt{1-\xi_i^2}$ are the ith undamped and damped modal frequencies, $\xi_i = \frac{c_i}{2\omega_i}$ is the corresponding damping ratio. The structural response of the ith mode can then be calculated through the convolution between the forcing function and the impulse response function as

$$z_i(t) = \int_0^t h_i(t-\tau)f_i(\tau)d\tau, \tag{7}$$