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Finite strain effects in piezoelectric energy harvesters under direct and parametric excitations

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ABSTRACT

This paper addresses the dynamic behavior of piezoelectric cantilevers under base excitations. Such devices are frequently used for applications in energy harvesting. An Euler-Bernoulli model that accounts for large-deflection effects and piezoelectric nonlinearities is proposed. Closed-form expressions of the frequency response are derived, both for direct excitation (i.e. with a base acceleration transverse to the axis of the cantilever) and parametric excitation (i.e. with a base acceleration along the axis of the cantilever). Experimental results are reported and used for assessing the validity of the proposed model. Building on the model presented, some critical issues related to energy-harvesting are investigated, such as the influence of nonlinearities on the optimal load resistance, the limits of validity of linear models, and hysteresis effects in the electrical power. The efficiency of direct and parametric excitation is also compared in detail.

1. Introduction

Energy harvesting from ambient vibrations has become an increasingly active topic in recent years [1–4]. The overall idea is to use ambient vibrational energy as a source for operating low-power electronic devices. Energy harvesters lie on a transduction mechanism that converts mechanical energy into electric energy. Among the various options that are available, piezoelectric materials – such as PZT ceramics – are often used. Those dielectric materials have a non symmetric crystalline structure that results in a natural polarization. Through the deformation of the crystalline lattice, deforming a piezoelectric material entails a change of the polarization, which in turn produces a variation of the electric field and a current flow in a surrounding electrical circuit. Such a behavior makes for very simple mechanical designs of energy harvesters: The cantilever configuration, represented in Fig. 1, is the most frequently used design [5–8]. It consists of a beam clamped on one side, with piezoelectric layers covering (all or part of) the top and bottom surfaces. Such a configuration is most commonly used in *direct*excitation, i.e. with a base acceleration that is transverse to the axis of the cantilever beam. The large majority of related studies uses a linear modeling approach, which rests on the underlying assumption that nonlinearities are sufficiently small to be neglected. Piezoelectric materials, however, have been reported to exhibit a nonlinear behavior, even at weak electric fields [9,10]. Assessing the validity of linear models thus needs to be clearly discussed, as pointed out notably in [11,12].

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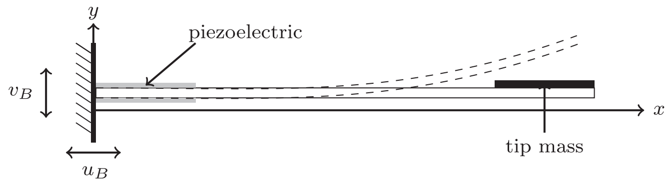

Fig. 1. Piezoelectric cantilever beam under base excitation.

Besides direct excitation, *parametric* excitation is an other way of using the piezoelectric cantilever as an energy harvester [13,14]. Parametric excitations is achieved by applying a base acceleration *along* the axis of the cantilever beam. In contrast with direct excitations, oscillations only occur if the excitation amplitude is sufficiently large. For excitations beyond that critical value, the amplitude of oscillations grows quadratically with the excitation amplitude (as far as nonlinear effects can be neglected) and can possibly reach large values [15]. Such a behavior raised the interest of using parametric excitations for energy harvesting [16,17]. Regarding piezoelectric cantilevers, parametric excitation has been far less studied than direct excitation. In particular, the comparison between direct and parametric excitations has not been fully addressed in the literature.

In order to address the issues mentioned above, we develop an Euler-Bernoulli model of piezoelectric cantilevers that accounts for large-deflection effects and piezoelectric elastic nonlinearities (Section 2). The predictions of that model are subsequently compared with experiments. A similar approach has been followed by Stanton et al. [18] for direct excitations, and by Daqaq and Stabler [13] for parametric excitations. Regarding direct excitations, Stanton et al. [18] took piezoelectric nonlinearities into account but ignored the geometric nonlinearities due to the large deflections. When comparing with experimental results, those authors needed to introduce a quadratic damping term to get a satisfactory match. But if the deflections are sufficiently large for quadratic damping to be significant, then there is no reason to neglect geometric nonlinearities in the first place. Geometric nonlinearities indeed give rise to a quadratic nonlinearity (through piezoelectric coupling). Regarding parametric excitations, Daqaq and Stabler [13] took geometric nonlinearities into account but ignored the piezoelectric nonlinearities. This might explain the noticeable differences obtained between the theoretical and experimental results in their paper.

The model developed in this paper accounts both for geometric and material nonlinearities. Abdelkefi et al. [14,19] proposed a distributed-parameter model of piezoelectric harvesters that includes both piezoelectric and geometric nonlinearities. They performed a parametric study to investigate the effects of all the nonlinear parameters introduced. Abdelkefi et al. also argued that 3 modes (at least) are needed in the Galerkin projection in most cases. Comparison with experiments, however, is not discussed in the works by Abdelkefi et al. [14,19]. Our modeling approach is similar to theirs. However, the comparison between the model and the experiments led us to make a different choice of material nonlinearities than those considered in [14]: We have taken a cubic term in the expression of the stress into account, and neglected some nonlinear piezoelectric coupling terms. Our expansion of the piezoelectric enthalpy is based on the assumption that, in the range of excitations considered, mechanical nonlinearities (large strain effects) dominate electric nonlinearities (large electric field effects).

Our purpose is to derive a model that captures the main physics of the problem but still remains simple enough for analytical investigation to be tractable and parameter identification to remain simple. To that end, we use a Galerkin projection on the linear mode whose frequency is the closest to the excitation frequency. This procedure leads to a 2-degrees of freedom dynamical system. Analytical expressions of approximate steady-state solutions are derived in Section 3 by using a multiple time scale expansion [20,21]. This enables us to obtain closed-form expressions of the frequency response, both for direct and parametric excitations. In Section 4 is reported an experimental study that allows the validity of the proposed model to be assessed. In Sect 5 we further explore some consequences of the model on some issues related to energy harvesting.

2. Large-deflection model of a cantilever piezoelectric beam

The most common configuration used in energy harvesting is represented in Fig. 1. It consists of a cantilever beam (of length L) subjected to base excitations. Piezoelectric patches cover a portion $[0, L_b]$ of the top and bottom surfaces of the beam. The goal of this Section is to derive a model of the piezoelectric-equipped beam that accounts for large deflections. In Section 2.1 we first study the purely mechanical problem of a heterogeneous beam. Specific features of the piezoelectric are introduced in Section 2.2. In particular, motivated by experimental observations, we consider nonlinear constitutive laws for the piezoelectric material.