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# High frequency, multi-axis dynamic stiffness analysis of a fractionally damped elastomeric isolator using continuous system theory

Luke Fredette, Rajendra Singh *

*Acoustics and Dynamics Laboratory, Smart Vehicle Concepts Center, Department of Mechanical and Aerospace Engineering, The Ohio State University, Columbus, OH 43210, USA*

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ABSTRACT

A spectral element approach is proposed to determine the multi-axis dynamic stiffness terms of elastomeric isolators with fractional damping over a broad range of frequencies. The dynamic properties of a class of cylindrical isolators are modeled by using the continuous system theory in terms of homogeneous rods or Timoshenko beams. The transfer matrix type dynamic stiffness expressions are developed from exact harmonic solutions given translational or rotational displacement excitations. Broadband dynamic stiffness magnitudes (say up to 5 kHz) are computationally verified for axial, torsional, shear, flexural, and coupled stiffness terms using a finite element model. Some discrepancies are found between finite element and spectral element models for the axial and flexural motions, illustrating certain limitations of each method. Experimental validation is provided for an isolator with two cylindrical elements (that work primarily in the shear mode) using dynamic measurements, as reported in the prior literature, up to 600 Hz. Superiority of the fractional damping formulation over structural or viscous damping models is illustrated via experimental validation. Finally, the strengths and limitations of the spectral element approach are briefly discussed.

## 1. Introduction

Elastomeric devices including vibration isolators are widely used in machines, equipment, and vehicles for their relatively low stiffness properties (with desired damping depending on the composition) at a relatively low cost. Prediction of their dynamic properties from basic principles is often difficult, since elastomeric materials often exhibit anisotropy, temperature- and age-dependence, as well as amplitude sensitive behavior [1–3]. As a result, dynamic stiffness at selected frequencies (and amplitudes of excitation under a given preload) are often measured using non-resonant elastomer test machines, though their operational bandwidth is often limited; even high frequency test machines are typically limited to around 500 to 1000 Hz in the uniaxial measurement mode [4,5]. However, the frequency range of interest may be much higher in many applications given vibration isolation, impedance mismatch, or acoustic comfort requirements [4–11]. Therefore, it is of vital importance to understand the isolator dynamics and its vibration transmission properties over a broad range of frequencies. Given the inherent limitations of elastomer test machines, few researchers have suggested

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* Corresponding author.
*E-mail address:* singh.3@osu.edu (R. Singh).

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**Nomenclature**

| | |
|---|---|
| $A$ | cross-sectional area |
| $a$ | left-side node (at $x = 0$) |
| $B$ | translational solution coefficient |
| $b$ | right-side node (at $x = L$) |
| $C$ | rotational solution coefficient |
| $c$ | viscous damping coefficient |
| $\mathrm{D}$ | generalized differential operator |
| $D$ | diameter |
| $E$ | elastic modulus |
| $F$ | force |
| $f$ | frequency (Hz) |
| $G$ | shear modulus |
| $H$ | shear equation coefficients |
| $h$ | loss factor |
| $I$ | second moment of area |
| $K$ | dynamic stiffness |
| $L$ | length |
| $M$ | moment |
| $N$ | number of terms |
| $Q$ | generalized force |
| $q$ | generalized displacement |
| $t$ | time |
| $u$ | translational displacement |
| $x, y, z$ | coordinates or directions |
| $\alpha$ | fractional damping order |
| $\beta$ | complex-valued wave number |
| $\gamma$ | $B$ to $C$ conversion factor |
| $\varepsilon$ | fractional damping coefficient |
| $\eta$ | fractional damping coefficient |
| $\theta$ | rotational displacement |
| $\kappa$ | Timoshenko shear coefficient |
| $\nu$ | Poisson's ratio |
| $\rho$ | density |
| $\omega$ | circular frequency (rad/s) |

*Subscripts*

| | |
|---|---|
| 0 | static |
| $e$ | excitation amplitude |
| $m, n$ | matrix indices (row, column) |
| max | maximum bandwidth |
| $\lambda$ | natural frequency |

*Abbreviations*

| | |
|---|---|
| DOF | degree(s) of freedom |
| FE | finite element |
| SEM | spectral element method |

indirect methods. In particular, Noll et al. [6] estimated the stiffness properties of elastomeric isolators with three degrees of freedom (DOF) embedded in an elastic beam system from the modal properties up to around 1 kHz; however, the isolator stiffness was assumed to be spectrally invariant. Kim and Singh [7] used mobility synthesis to estimate the multi-axis dynamic properties of an isolator between two known mass elements by measuring the frequency responses of the assembled structure up to 2 kHz. Finally, Meggitt et al. [8] proposed a similar procedure to determine *in situ* isolator properties up to 2 kHz. In each case, the frequency range is restricted by experimental limitations and the underlying estimation methods.

Typically, the static and dynamic analyses in industry are conducted with commercial finite element (FE) codes. This approach has some disadvantages since many elements are required to yield accurate predictions of dynamic properties, particularly at higher frequencies. Component-level analysis of an elastomeric joint may not accurately capture its influence on a larger system, but embedding multiple high-order, FE joint type models into a larger system model can escalate the model size to an unreasonable extent. Furthermore, the usable frequency range of modal or dynamic stiffness analyses of such structures is limited by the element size, parameter uncertainty, and natural frequency spacing, suggesting the need for an analytical approach that may guide computationally intensive exercise in a more rational manner. For instance, minimal order lumped parameter models have been considered to address these difficulties with partial success over the lower frequency range [3,7,9]. However, the inertial and elastic properties are more distributed at higher frequencies or smaller wavelengths [10]. This suggests the use of continuous system methods though only certain types of solutions are tractable [11,12]. In particular, Kim and Singh [11] applied the continuous theory to flexural and longitudinal motion of an isolator, predicting the dynamic properties of elastomeric paths up to 4 kHz with the structural damping assumption. Likewise, Östberg and Kari [12] proposed a wave-guide approach for fractionally damped cylindrical isolators which achieved accurate dynamic stiffness predictions as long as the isolator length is much larger than its diameter.

This article seeks to address some of the above mentioned limitations by employing the spectral element method (SEM) to develop a coupled, 6-DOF dynamic stiffness matrix of a fractionally damped viscoelastic isolator (with a relatively short aspect ratio) up to 5 kHz, that would extend beyond the typical measurement range. The proposed method intends to offer analytical solutions that should supplement computationally expensive high-fidelity FE predictions.

## 2. Problem formulation

The spectral element method [13] in some ways combines features of both continuous system and minimal order models. Minimal order models have the advantage of dramatically decreased computation time, enhanced physical insight