Journal of Sound and Vibration ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Robust partial quadratic eigenvalue assignment with time delay using the receptance and the system matrices ☆

Zheng-Jian Bai [a,b,*], Jin-Ku Yang [a], Biswa Nath Datta [c]

[a] School of Mathematical Sciences, Xiamen University, Xiamen 361005, People's Republic of China
[b] Fujian Provincial Key Laboratory of Mathematical Modeling & High Performance Scientific Computing, Xiamen University, Xiamen 361005, People's Republic of China
[c] Department of Mathematical Sciences, Northern Illinois University, DeKalb, Illinois 60115, USA

ARTICLE INFO

Article history:
Received 14 January 2016
Received in revised form 24 June 2016
Accepted 1 August 2016
Handling Editor: J. Lam

Keywords:
Partial quadratic eigenvalue assignment
robustness
receptance measurements
time delay

ABSTRACT

In this paper, we consider the robust partial quadratic eigenvalue assignment problem in vibration by active feedback control. Based on the receptance measurements and the system matrices, we propose an optimization method for the robust and minimum norm partial quadratic eigenvalue assignment problem. We provide a new cost function and the closed-loop eigenvalue sensitivity and the feedback norms can be minimized simultaneously. Our method is also extended to the case of time delay between measurements of state and actuation of control. Numerical tests demonstrate the effectiveness of our method.

1. Introduction

The dynamics of vibrating structures modeled by systems of matrix second-order differential equations of the form:

$$M\ddot{\mathbf{x}}(t)+C\dot{\mathbf{x}}(t)+K\mathbf{x}(t)=\mathbf{f}(t) \tag{1}$$

are governed by the eigenvalues and eigenvectors of the associated quadratic matrix pencil [46]

$$P(\lambda)\equiv\lambda^2M+\lambda C+K,$$

where the real $n\times n$ matrices M, C, K are mass, damping and stiffness matrices respectively. The eigenvalues are related to the natural frequencies and the eigenvectors are just the mode shapes [3,5,23,26].

When a natural frequency becomes equal or close to the external frequency of force $\mathbf{f}(t)$, the structure is subjected to dangerous vibration due to the resonance.

Though passive damping technique is economic and thus widely used in practice, it has severe practical limitations. On the other hand, the Active Vibration Control (AVC) technique is more scientific in nature and large complex structures such as high-rise buildings and bridges in several countries have been built based on this technique.

☆ The research of this author is partially supported by the National Natural Science Foundation of China (No. 11671337), the Natural Science Foundation of Fujian Province of China (No. 2016J01035), and the Fundamental Research Funds for the Central Universities (No. 20720150001).

* Corresponding author at: School of Mathematical Sciences, Xiamen University, Xiamen 361005, People's Republic of China.
E-mail addresses: zjbai@xmu.edu.cn (Z.-J. Bai), yangjinku163@163.com (J.-K. Yang), dattab@math.niu.edu (B.N. Datta).

http://dx.doi.org/10.1016/j.jsv.2016.08.002

The most important and difficult aspect of implementation of AVC technique is to effectively compute two feedback matrices, such that a small number of resonant eigenvalues are eliminated from the structure leaving other large number of eigenvalues and eigenvectors unchanged.

Let $\{\lambda_i, \mathbf{x}_i\}_{i=1}^{2n}$ be the $2n$ eigenvalues and eigenvectors of the pencil $P(\lambda)$ and

$$\mathbf{f}(t) = B\mathbf{u}(t), \quad \mathbf{u}(t) = F^T\dot{\mathbf{x}}(t) + G^T\mathbf{x}(t), \tag{2}$$

where B is a given real n-by-m control matrix, $\mathbf{u}(t)$ is the associated control m-vector, and the real n-by-m matrices F and G are called feedback matrices. Then the problem is to construct the feedback matrices F and G such that the spectrum of the closed-loop pencil

$$P_c(\lambda) \equiv \lambda^2 M + \lambda(C - BF^T) + (K - BG^T)$$

is $\{\mu_1, \ldots, \mu_p, \lambda_{p+1}, \ldots, \lambda_{2n}\}$, where $\mu_1, \ldots, \mu_p$ $(p \ll 2n)$ are the set of suitably chosen numbers that replace the resonant eigenvalues $\lambda_1, \ldots, \lambda_p$. In addition, the eigenvectors $\mathbf{x}_{p+1}, \ldots, \mathbf{x}_{2n}$ corresponding to the no spill-over eigenvalues $\lambda_{p+1}, \ldots, \lambda_{2n}$ are required to remain unchanged.

The problem of finding F and G this way is known as the Partial Quadratic Eigenvalue Assignment Problem (PQEAP). We note that since the mass matrix M is not updated and is assumed to be symmetric and positive definite, the closed-loop eigenvalues of $P_c(\lambda)$ are all finite (see [14, Section 11.9]).

A practical challenge is to solve the problem in second-order setting itself without transforming it to a standard first-order control problem and to solve it using only a small amount of eigenvalues and eigenvectors that can be computed or measured in practice. Furthermore, the aspect of no spill-over must be established by means of mathematical theory. Such direct solution approach was first proposed in 1997 by Datta et al. [15] for the single-input case. Since then, a large number of papers have been published both for the single-input and the multi-input solution of the PQEAP. See for instance [10,16–19,37].

Furthermore, practical aspects of minimizing feedback norms and robust stabilization have been considered and solved using techniques of numerical optimization (e.g., [2,6,7,9,11,13,27–29,36]). The solution methods in these works were based on the knowledge of system matrices M, C, and K, but did not take advantage of the receptances, which are freely available in practice by measurements.

Recently, the measured receptances have been used in AVC (e.g., [31–33,38,39,42–45]). Hybrid approaches, combining both system matrices and receptances, however, have been developed in the last few years only for the single-input PQEAP, the multi-input PQEAP, or the feedback norm minimization [1,40].

In this paper, we propose an optimization-based hybrid approach for simultaneous norm- minimization and robustness of the closed-loop system. Mathematically, the robustness problem is to construct F and G in such a way that the condition number of the closed-loop eigenvalue is minimized. We then extend our solution technique to the time-delay system.

The main contributions of the paper are as follows:

A new optimization algorithm is proposed for simultaneous minimization of the feedback norms and the condition number of the closed-loop system of the PQEAP arising in AVC, both for time-invariant and time-delay systems.

(i) The algorithm is direct in the sense that it does not require transformation in the standard first-order system.
(ii) It is implementable using any small numbers of frequencies and mode shapes that are measurable at a vibration laboratory or computable using state-of-the-art computational techniques.
(iii) It uses knowledge of both the system matrices and the receptances, which are readily available by measurements.
(iv) It can exploit the inherent physical structures of the problem, such as, the symmetry, positive definiteness, and sparsity of the matrices M, C, and K, which are often assets for large-scale computations.

We provide illustrative numerical examples to demonstrate the effectiveness of the proposed algorithms and comparisons are made with the existing algorithms whenever appropriate.

The results of comparisons show that the hybrid algorithms, combining the knowledge of both system matrices and the measured receptances, were better in almost all cases considered.

2. Preliminaries

In what follows, we assume that M, C and K are real symmetric with M being positive definite. Let I be the identity matrix of appropriate dimension. Suppose that $\{\mu_1, \ldots, \mu_p\} \cap \{\lambda_1, \ldots, \lambda_{2n}\} = \varnothing$ and $\{\lambda_1, \ldots, \lambda_p\} \cap \{\lambda_{p+1}, \ldots, \lambda_{2n}\} = \varnothing$. The control matrix B has full column rank, and $(P(\lambda), B)$ is partially controllable with respect to the eigenvalues $\lambda_1, \ldots, \lambda_p$, i.e.,

$$(P(\lambda_i), B) = n, \quad i = 1, \ldots, p.$$

Let

$$\Lambda_1 = \mathrm{diag}(\lambda_1, \ldots, \lambda_p), \quad \Lambda_2 = \mathrm{diag}(\lambda_{p+1}, \ldots, \lambda_{2n})$$

Please cite this article as: Z.-J. Bai, et al., Robust partial quadratic eigenvalue assignment with time delay using the receptance and the system matrices, *Journal of Sound and Vibration* (2016), http://dx.doi.org/10.1016/j.jsv.2016.08.002