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# Bayesian analysis of the flutter margin method in aeroelasticity

Mohammad Khalil [a,1], Dominique Poirel [b], Abhijit Sarkar [a,*]

[a] Carleton University, Department of Civil and Environmental Engineering, Mackenzie Building, Colonel By Drive, Ottawa, Ontario, Canada K1S 5B6

[b] Department of Mechanical and Aerospace Engineering, Royal Military College of Canada, Kingston, Ontario, Canada K7K 7B4

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ABSTRACT

A Bayesian statistical framework is presented for Zimmerman and Weissenburger flutter margin method which considers the uncertainties in aeroelastic modal parameters. The proposed methodology overcomes the limitations of the previously developed least-square based estimation technique which relies on the Gaussian approximation of the flutter margin probability density function (pdf). Using the measured free-decay responses at subcritical (preflutter) airspeeds, the joint non-Gaussain posterior pdf of the modal parameters is sampled using the Metropolis–Hastings (MH) Markov chain Monte Carlo (MCMC) algorithm. The posterior MCMC samples of the modal parameters are then used to obtain the flutter margin pdfs and finally the flutter speed pdf. The usefulness of the Bayesian flutter margin method is demonstrated using synthetic data generated from a two-degree-of-freedom pitch-plunge aeroelastic model. The robustness of the statistical framework is demonstrated using different sets of measurement data. It will be shown that the probabilistic (Bayesian) approach reduces the number of test points required in providing a flutter speed estimate for a given accuracy and precision.



## 1. Introduction

In aeroelastic flutter predictions, the oldest and classic methodology involves tracking damping against airspeed or dynamic pressure whereby the flutter speed is identified as the critical airspeed reaching zero damping [1]. The major obstacles this technique faces are the sensitivity of damping estimates against the noise inherent in the observational data and lack of monotonic decay of damping against airspeed. These difficulties have most likely motivated the development of the so-called flutter margin method [2]. While strictly applicable to coalescence flutter, the flutter margin, expressed using both aeroelastic frequencies and damping of the two modes participating in flutter, exhibits monotonic decay against airspeed [2]. Furthermore, due to the inherent physics of coalescence flutter, this method is much more sensitive to frequency than to damping [1]. This is an additional benefit of the flutter margin method since modal frequencies can be estimated with less uncertainty than damping.

The current investigation specifically focusses on the effect of uncertainty on coalescence flutter in aeroelasticity. An extensive body of literature exists for uncertainty quantification in aeroelasticity (for example, see [3–7]). In the context of coalescence flutter, Zimmerman and Weissenburger [2] defined the flutter margin as an indicator of the stability of an

* Corresponding author. Fax: +1 613 520 3951.
*E-mail address:* abhijit_sarkar@carleton.ca (A. Sarkar).
[1] Currently at Sandia National Laboratories, P.O. Box 969 MS 9051, Livermore, CA 94551-9051, United States.

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aeroelastic system undergoing coalescence flutter using Routh's criteria. For quasi-steady aerodynamics, the flutter margin is defined as the third subdeterminant of the fourth-order characteristic equation of the two-degree-of-freedom aeroelastic system as dictated by Routh's criteria [2]. For stable aeroelastic systems, the flutter margin is a positive quantity defined by the modal frequencies $\omega_1$, $\omega_2$ and the decay rates $\beta_1$, $\beta_2$ of the aeroelastic modes participating in the flutter instability [2]

$$F = \left[\left(\frac{\omega_2^2-\omega_1^2}{2}\right)+\left(\frac{\beta_2^2-\beta_1^2}{2}\right)\right]^2 + 4\beta_1\beta_2\left[\left(\frac{\omega_2^2+\omega_1^2}{2}\right)+2\left(\frac{\beta_2+\beta_1}{2}\right)^2\right] - \left[\left(\frac{\beta_2-\beta_1}{\beta_2+\beta_1}\right)\left(\frac{\omega_2^2-\omega_1^2}{2}\right)+2\left(\frac{\beta_2+\beta_1}{2}\right)^2\right]^2. \tag{1}$$

The flutter margin can be used to extrapolate the flutter speed using measured flight test responses gathered at sub-critical airspeeds. Under the assumption of quasi-steady aerodynamics and in the absence of structural damping, the flutter margin for a 2-DOF typical section model varies quadratically with dynamic pressure [2]. For unsteady aerodynamics, the relationship is more complex which may be captured by higher order polynomials in dynamic pressure or equivalently airspeed [8]. For example, the consideration of structural damping in the analysis leads to a sixth-order relationship between the flutter margin and airspeed as demonstrated by Price and Lee [9]. In practice, one may not know the complexity of this relationship prior to the analysis of flight test data. Regarding the added complexity due to structural damping, as the airspeed increases, the aerodynamic damping controls the total damping. In this situation, the flutter margin can be approximated by a second-order polynomial in dynamic pressure as [2]

$$F = B_2q^2 + B_1q + B_0. \tag{2}$$

As suggested by Zimmerman and Weissenburger [2], and pointed out by Bennett [8], the sensitivity of the flutter margin extrapolation to the $B_2$ parameter is such that unless it is estimated accurately, it is preferable to use a linear extrapolation in dynamic pressure or a second-order polynomial in airspeed, as per Eq. (3). The coefficients, $b_1$ and $b_2$, can be estimated using a least-square fitting procedure of Eq. (3) with at least two sub-critical airspeeds. The flutter speed, $U_f$, is defined as the positive real root of Eq. (3):

$$F = b_1 + b_2U^2. \tag{3}$$

In practice, reduced forms of the flutter prediction equation are used such as parabolic and/or linear extrapolations in airspeed; see the F-15 [10] and the F-18 [11] flight flutter test programs, respectively.

A number of extensions and generalizations of the flutter margin method have been reported following the original work of Zimmerman and Weissenburger [2]. Price and Lee [9] reported the extension of this method for the case of trinary flutter. Based on Jury's stability analysis, which is the equivalent of Routh's stability for analog systems, a corresponding flutter margin method for discrete-time systems was developed by Torii and Matsuzaki [12]. Poirel et al. [1] considered the modal parameter uncertainties in order to obtain the flutter margin pdf and the corresponding flutter speed pdf. In their investigation, the uncertainties in the modal parameters are propagated to obtain the histograms of the flutter margins and flutter speed from which various statistical quantities of interest are estimated. Despite the Gaussian approximation of the modal parameter, both the flutter margin and flutter speed pdfs turned out to be non-Gaussian as illustrated from F-18 flight test data in Poirel et al. [1]. Heeg [13] carried out a further examination of the methodology by Poirel et al. [1] whereby the maximum likelihood parameter estimates are used to construct the flutter speed pdf. Abbasi and Cooper [14] investigated the statistical pdf of the flutter margin concluding that for low values of modal parameter uncertainty the flutter margin was nearly Gaussian, and became skewed as the noise level increased (to levels generally observed in flight tests) and when the two frequencies became close (as is typical of coalescence flutter).

The main contribution of this paper lies on the detailed statistical formulation of the flutter margin method using the Bayesian formalism and demonstration of the robustness of this Bayesian methodology using a two-degree-of-freedom pitch-plunge aeroelastic system. In our initial attempts, Khalil et al. [15–17] built on Poirel et al. [1] proof-of-concept and provided a pdf of the flutter speed by estimating the joint pdf of the flutter prediction equation coefficients by applying a Bayesian inference methodology. The flutter margin pdfs were estimated from modal parameters corrupted by Gaussian noise. Extending the basic methodology developed by Khalil et al. [15–17] for aeroelastic system with quasi-steady aerodynamics, this paper includes the effects of unsteady aerodynamics and structural damping to further demonstrate the usefulness of the Bayesian framework. The modeling error arising from unsteady aerodynamics is also inferred within the Bayesian setting. Firstly, the aeroelastic frequencies, damping and modeling error strength are inferred using the MH MCMC sampling from the noisy free-decay response of the system. Secondly, the MH MCMC sampling is carried out again to estimate the parameters of the flutter margin equation (3). Furthermore, the robustness of the Bayesian analysis is studied against the number of tests conducted at different airspeeds. The proposed method is illustrated using coupled pitch and plunge oscillations of an aeroelastic system with known flutter speed. It will be shown that the probabilistic (Bayesian) approach reduces the number of test points required in providing a flutter speed estimate for a given accuracy and precision. The preliminary findings of the paper are reported in a number of conferences [15–20].