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Parametric study of the mode coupling instability for a simple system with planar or rectilinear friction

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ABSTRACT

In this paper, the study of a damped mass–spring system of three degrees of freedom with friction is proposed in order to highlight the differences in mode coupling instabilities between planar and rectilinear friction assumptions. Well-known results on the effect of structural damping in the field of friction-induced vibration are extended to the specific case of a damped mechanical system with planar friction. It is emphasised that the lowering and smoothing effects are not so intuitive in this latter case. The stability analysis is performed by calculating the complex eigenvalues of the linearised system and by using the Routh–Hurwitz criterion. Parametric studies are carried out in order to evaluate the effects of various system parameters on stability. Special attention is paid to the understanding of the role of damping and the associated destabilisation paradox in mode-coupling instabilities with planar and rectilinear friction assumptions.

1. Introduction

Even if the problem of friction-induced vibrations has been a topic of great interest for many researchers in the past years [1–6], squeal is still a current issue and a difficult subject since the effects of the system parameters are not completely understood. The comprehension of the phenomena involved in these vibrations and the choice of the contact models at the frictional interface have been much investigated [2–4,7] and are still an active field of research. For example, Massi et al. [8] highlight the significant impact of both the contact surface topography and the frictional contact model. Many mechanisms have been proposed to explain the emergence of friction-induced instabilities. It is generally assumed that four categories of friction-induced vibration may be distinguished [4,7]: stick-slip, variable dynamic friction coefficient, sprag-slip and mode coupling. Stick-slip was the first developed theory. Similarly, a decrease of the friction coefficient with relative sliding velocity (which introduces a negative damping due to tribological properties) can be chosen as the source of instability. In this case, a single structural mode can become unstable. Next, academic studies demonstrated that self-excited vibration can occur with a constant friction coefficient. Spurr [9] introduced the well-known sprag-slip phenomenon based on a geometrically induced instability. Then, several authors proposed the mode coupling mechanism as the origin of instability. In this latter case, the instability is the result of both the coupling of at least two structural modes of the mechanical system

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and the coupling between the normal and the tangential dynamics. It is associated to a Hopf bifurcation. This approach is now widely used to reproduce friction-induced vibrations in mechanical systems. Minimal models highlighting this type of instabilities can be easily found in the literature [10–12]. In most of these studies, the contact between the mass and the friction plane is modelled by a classical contact spring. It is a relatively common choice not only for this type of minimal models but also for most elaborated finite element models. This choice can be explained by physical considerations (i.e. local elasticity of the contact) or by numerical considerations (i.e. the friction force is directly linked to the normal displacement in order to simplify the problem formulation and the equations of motion). In addition, the authors often restrained the contact formulation at the frictional interface to one tangential degree-of-freedom (i.e. rectilinear friction). However, in this case, some specific phenomena due to the planar nature of the friction Coulomb law cannot be explained, particularly the fact that the direction of the friction force varies with the structure vibration.

A simple model combining these two characteristics (i.e. no contact spring and planar friction) was proposed by Moirot [13,14]. He outlined that differences between planar and rectilinear friction formulations can be highlighted and that destabilisation due to the planar friction may occur. However, Moirot's study focused only on a system without damping. On the contrary, similar studies carried out on more complex finite element models [15–17] showed that planar friction seems to stabilise the system.

Other planar effects of friction have already been investigated in the general context of friction-induced vibrations or brake squeal. Kinkaid et al. [18] proposed a new mechanism for disc brake noise by taking into account both longitudinal and transverse braking directions for a 4-dof model. They demonstrated that instabilities in the radial vibration can be observed during the transient processes. Oberst and Lai [19] also studied nonlinear friction coupling in disc brake squeal and more specifically the influence of the lateral vibration of a 2-dof friction oscillator. Considering the variation of the belt angle, they showed that a perturbation in the radial component in a brake system can also cause instabilities. Zhang et al. [20] recently studied the stability of coupled friction oscillators on sliding rigid plate with planar friction. They investigated the influence of the uncertainty arising from the tribological aspect and proposed different types of friction modelling with randomised parameters.

However, the precise role of planar friction on mode-coupling instabilities, compared with rectilinear friction, remains an open question. The objective of the present study is to investigate this scientific issue thoroughly. One of the most interesting phenomena concerns the existence of a non-intuitive effect of damping distribution that can generate unstable vibrations. As previously reported in many recent works, structural damping is of primary importance to mode coupling stability and results indicate that the addition of damping in mechanical systems alters the stable–unstable boundaries. In the literature, the relationship between damping and system propensity to develop instability and the so-called "destabilisation paradox" applied to friction-induced vibration have often been illustrated by several analytical and numerical studies [12,21–23]. Many contributions on the destabilisation paradox in conservative systems with damping have also been carried out in a general context by Kirillov [24–27]. He showed that the effect of proportional and non-proportional damping on the reversible Hopf bifurcation in systems with rectilinear friction is in qualitative agreement with the general theory of the destabilisation paradox in circulatory systems [25–27]. Massi and Giannini [28] also proposed an experimental investigation of the relationship between the distribution of modal damping and the propensity to develop squeal in a beam-on-disk setup. They highlighted that a nonuniform repartition of the modal damping causes an increase of the squeal propensity. Extensive studies have also been conducted including the role of damping with other physical factors. For example Hervé et al. [29,30] used a two-degree-of-freedom nonlinear model of clutch squeal in order to examine in detail the influence of structural damping on the effects of the combined circulatory and gyroscopic actions. Recently, Kirillov et al. [25–27] worked on the determination in an explicit form of the stabilising damping configurations for a large class of non-conservative systems. They proposed a theory for the qualitative and quantitative description of the destabilisation paradox in such systems. In the case of a vibrational system, planar friction introduces a new contribution of damping which has not been examined in this perspective. The additional inclusion of structural damping also remains an open question. Thus, an original contribution of the present study is to extend well-known results on the non-intuitive effects of damping distribution for the specific case of a damped mechanical system with planar friction.

First, the mechanical system under study, the background on stability analysis and the Routh–Hurwitz criterion are presented. Second, parametric studies and numerical results for damped and undamped systems with planar or rectilinear friction are investigated in order to discuss the elementary effect of a planar or rectilinear friction and to undertake the extension of the destabilisation paradox in the presence of planar friction.

2. Description of the mechanical model

The system considered in this study is described in Fig. 1. It is composed of a mass m in frictional contact with a rigid plane moving with a constant rectilinear velocity. The value of the imposed velocity is denoted V while its direction is denoted $\vec{t}$ as indicated in Fig. 1. The mass is held against the moving plane by three springs and pressed by an external force F. Damping is also included as shown in Fig. 1. The angle between the direction $\vec{t}$ of the imposed velocity of the rigid plane and the coordinate $\vec{x}$ of the mechanical system is defined by θ. The equations of motion can be written in matrix