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Cable dynamics under non-ideal support excitations: Nonlinear dynamic interactions and asymptotic modelling

Tieding Guo *, Houjun Kang, Lianhua Wang, Yueyu Zhao

College of Civil Engineering, Hunan University, Changsha, Hunan 410082, PR China

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ABSTRACT

Cable dynamics under ideal longitudinal support motions/excitations assumes that the support's mass, stiffness and mechanical energy are infinite. However, for many long/slender support structures, their finite mass and stiffness should be taken into account and the cable-support dynamic interactions should be modelled and evaluated. These moving supports are non-ideal support excitations, deserving a proper coupling analysis. For systems with a large support/cable mass ratio, using the multiple scale method and asymptotic approximations, a cable-support coupled reduced model, with both cable's geometric nonlinearity and cable-support coupling nonlinearity included, is established asymptotically and validated numerically in this paper. Based upon the reduced model, cable's nonlinear responses under non-ideal support excitations(and also the coupled responses) are found, with stability and bifurcation characteristics determined. By finding the modifications caused by the support/cable mass ratio, boundary damping, and internal detuning, full investigations into coupling-induced dynamic effects on the cable are conducted. Finally, the approximate analytical results based on the reduced model are verified by numerical results from the original full model.

1. Introduction

Cables used in suspension/stay cable bridges, power transmission systems, marine mooring systems, are important tension structural members. Cable's dynamic behaviors under external or support excitations are complicated and interesting. Therefore, cable dynamics has attracted many researchers' attentions in the past few decades [1,2]. Cable's linear dynamic model and modal results were developed by Irvine [3,4] and detailed dynamic analysis on mooring cables was conducted by Triantafyllou [5]. Elastic cable's nonlinear oscillation was investigated by Luongo [6]. Based upon these work, cable dynamics has been extensively investigated by many researchers in the past few decades [7–16].

Besides external excitations directly on the cables, cable's support motions would induce kinetic excitations, or termed as support excitations, which are also very important for cables in structural engineering. For example, power tower's motion for transmission cables, bridge tower's motion for suspended cables, bridge deck's motion for the inclined cables, and floating platform's motion for the mooring cables. In this respect, Perkins [7] formulated suspended cable's mode interactions under longitudinal/tangential support's motion, and Benedettini [17] developed a discrete cable model under

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* Corresponding author.
E-mail address: guotd@hnu.edu.cn (T. Guo).

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vertical/out-plane support motions. Thereafter, stay/inclined cable's nonlinear vibrations caused by bridge deck/tower's motions have been investigated by many researchers [18–27].

We term the above support motions as ideal support excitations, as the cable's reactions on the support are neglected. Essentially, the support is assumed to be of infinite stiffness, inertia, and energy. This is ideal in physics, and the assumption has got challenged recently by more and more long/high-rise flexible tower/support structures, which have been used in suspension/stay cable bridges and power transmission systems. Explicitly, the slender support's own finite flexibility and inertia should be carefully modelled and dynamically evaluated. We term these support motions as non-ideal support excitations. This is equivalent to say that one should investigate cable-flexible support dynamic interactions [12–16,28–33] for further understanding of more realistic support excitations. For example, by introducing local and global vibrations into the stayed cable-beam system, a cable-support coupled model was proposed in Ref. [12]. Furthermore, in Refs. [13,14], based upon a more advanced cable-beam model(with a strict cable-beam coupled eigen-problem solved), the local and global modes were analytically derived and complex cable-beam coupled dynamics including mode interactions were investigated.

The cable-support coupled modelling/analysis is complicated and the previous investigations [12–16,28–33] are very enlightening. We note that in most engineering cable systems, the support/cable mass ratio is a very large parameter. We can reduce the difficulties of coupled modelling/analysis significantly if confining ourselves to a cable-support coupled model valid in an asymptotic sense. Therefore, our first aim is to establish a rational asymptotic formulation for cable dynamics under non-ideal support excitations, through asymptotic modelling of cable-support nonlinear dynamic interactions. And furthermore, our second aim is to investigate the coupling/dynamic interactions induced dynamic effects on cables.

This paper is organized as follows. In Section 2, a brief statement of cable-support nonlinear dynamic interactions is presented. Section 3 is devoted to cable-support system's asymptotic expansions using the multiple scale method, with a reduced coupled model established. After numerical validations of the reduced model in Section 4.2, detailed nonlinear responses and coupling-induced dynamic effects are analyzed in Sections 4.3 and 4.4. Conclusions are given in Section 5.

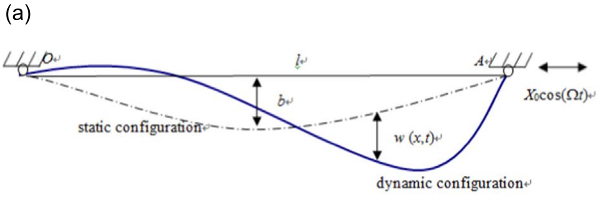
2. Problem's full formulation

A suspended elastic cable is fixed at support O, and coupled to a longitudinal oscillating support at A, as depicted in Fig. 1, including ideal and non-ideal support motions/excitations in (a) and (b), respectively. For the ideal case, the support's mass and stiffness are infinite(not shown in the figure). For the non-ideal case, the support's large(finite) mass and stiffness are denoted by M and K, respectively.

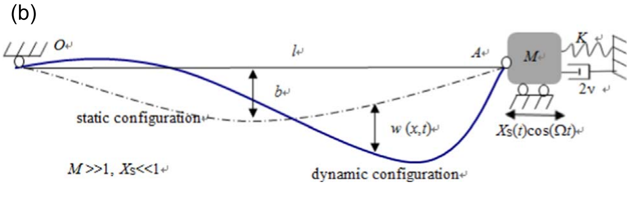
Assuming that the cable stretches in a quasi-static manner, we have the following non-dimensional governing equation for cable's in-plane vertical dynamics under longitudinal support motion [22]:

$$\ddot{w}+2c\dot{w}-w''-\alpha(w''+y'')\left[s(t)+\int_0^1\left(y'w'+\frac{1}{2}w'^2\right)dx\right]=0 \tag{1}$$

where $w(x,t)$ is cable's in-plane vertical displacement and $s(t)$ is support's longitudinal displacement. Cable's non-dimensional stiffness is $\alpha=EA/H=8bEA/m_c gl^2$, where the Young's modulus, area of the cross section, cable tension's horizontal component, cable's mass per unit length, and the gravitational acceleration are represented by E, A, H, m_c, and g respectively. The sag-to-span ratio is $f=b/l$ and cable's non-dimensional configuration function is $y(x)=4fx(1-x)$, where b is



A suspended cable under ideal support excitations



A suspended cable under non-ideal support excitations

Fig. 1. Cable-support dynamic interactions: ideal vs. non-ideal support excitations.

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