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Modal identification using the frequency-scale domain decomposition technique of ambient vibration responses

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ABSTRACT

This paper proposes a new modal identification method of ambient vibration responses. The application of the singular value decomposition to continuous wavelet transform of power spectral density matrix gives singular values and singular vectors in frequency-scale domain. Analytical development shows a direct relation between local maxima in frequency-scale representation of singular values and modal parameters. This relation is then carried on for the identification of modal parameters via a complete practical procedure. The main novelties of this work involve the new formulation in frequency-scale domain and the capacity for the identification of modal parameters without the step of ridges extraction in comparison with previous wavelet-based modal identification methods.

1. Introduction

The structures's modal properties (i.e., natural frequencies, damping ratios and mode shapes) are commonly used for dynamic analysis, structural health monitoring, updating finite-element models, detecting structural modification, and so on [1]. Dynamic tests usually provide these modal properties. One of the most convenient techniques for real structures is ambient vibration testing, in which the excitation is of natural form, such as wind, traffic, or waves. Because of this natural excitation source, ambient vibration tests have advantages over impulse and periodic excitation, such as low cost, broad-band excitation, and continuous use of the structures. Ambient excitation, however, cannot be controlled or measured, hence the name “unknown input” and the need for special processing to obtain the modal parameters.

Modal analysis techniques for ambient vibration testing are called operational modal analysis (OMA) and they can be classified by signal processing domain (time, frequency or time–frequency) or by model-based methods (parametric or non-parametric). Several techniques are available for OMA, such as autoregressive moving average (ARMA model: time/parametric method) [2,3], stochastic subspace identification (SSI: time/parametric method) [4], enhanced frequency domain decomposition method (EFDD: frequency/non-parametric method) [5,6], continuous wavelet transform (CWT: time–frequency/non-parametric) [7–12].

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The application of wavelet transform to ambient vibration responses for modal identification has begun about two decades ago. Ruzzene et al. [8], Lardies and Gouttebroze [9], and Slavic et al. [10] proposed the processing by two steps: free responses are first obtained from ambient responses by applying the random decrement technique; and then, the wavelet transform is applied to these free responses for modal parameters extraction. Le and Paultre [11] identified modal parameters using direct application of the wavelet transform to ambient vibration responses via correlation function matrix. Recently, the time–frequency domain decomposition (TFDD) method was proposed by Le and Paultre [12] based on the continuous wavelet transform and singular value decomposition algorithm. Tarinejad and Damadipour [13] combined the frequency domain decomposition method and the continuous wavelet transform method based on cyclic averaging of correlation functions for identification of dynamics properties. In order to extend its application for different kinds of structures, especially for huge structures [14], they proposed to correct the errors resulting from asynchronous sensing of sensors.

In all above references, the wavelet transform was applied to responses in time domain. Argoul [15], Yin and Argoul [16] and Yin et al. [17] used the continuous wavelet transform for a frequency response function of input–output dynamic tests. They identified natural frequencies and damping ratios but not mode shapes. This paper describes an exploration when the wavelet transform is applied to ambient vibration responses in frequency domain via their power spectral density (PSD) matrix. After a brief background of ambient vibration responses and continuous wavelet transform in Section 2. Section 3 focuses on the development of a new modal identification method. From analytical formulations, a practical procedure of the method is deduced. In Section 4, the proposed method is first applied to numerical examples and then to a laboratory test. The identified results are compared to exact values or to those derived from an established technique to show its validity.

2. Background for ambient responses and continuous wavelet transform

2.1. Ambient vibration responses

A N degree-of-freedom linear mechanical system can be characterized by a modal model comprising N natural angular frequencies ω_k, damping ratios ξ_k and mode shapes $\boldsymbol{\phi}_k$ $(k=1\ldots N)$. Under ambient excitation, in general modeled by a zero mean Gaussian white noise process, responses of the system $\mathbf{x}(t)$ are also Gaussian processes with zero mean [18–20]. The responses $\mathbf{x}(t)$ can be displacement, velocity or acceleration.

The correlation function matrix $\mathbf{R}_{\mathbf{xx}}(\tau)$ and its Fourier transform matrix called power spectral density matrix $\hat{\mathbf{R}}_{\mathbf{xx}}(\omega)$ are expressed by [12]:

$$\begin{aligned}\mathbf{R}_{\mathbf{xx}}(\tau) &= \sum_{k=1}^{N} \boldsymbol{\phi}_k e^{\lambda_k \tau} \mathbf{q}_k^T + \overline{\boldsymbol{\phi}}_k e^{\overline{\lambda}_k \tau} \overline{\mathbf{q}}_k^T \\ \hat{\mathbf{R}}_{\mathbf{xx}}(\omega) &= \sum_{k=1}^{N} \frac{\boldsymbol{\phi}_k \mathbf{q}_k^T}{i\omega - \lambda_k} + \frac{\overline{\boldsymbol{\phi}}_k \overline{\mathbf{q}}_k^T}{i\omega - \overline{\lambda}_k}\end{aligned} \tag{1}$$

where $\boldsymbol{\phi}_k$, λ_k are respectively the mode shape and the pole of mode k and $\mathbf{q}_k$ is a vector whose components are constant. The relation between λ_k and natural angular frequency ω_k and damping ratio ξ_k is: $\lambda_k = -\xi_k\omega_k + i\omega_k\sqrt{1-\xi_k^2} = -\xi_k\omega_k + i\tilde{\omega}_k$. The symbols $\overline{(.)}$ and $(.)^T$ denote respectively conjugate and transpose operators.

Under assumption of white noise excitation and light damping, the power spectral density matrix can be written by [5,13]:

$$\hat{\mathbf{R}}_{\mathbf{xx}}(\omega) = \sum_{k=1}^{N} \frac{d_k \boldsymbol{\phi}_k \boldsymbol{\phi}_k^T}{i\omega - \lambda_k} + \frac{\overline{d}_k \overline{\boldsymbol{\phi}}_k \overline{\boldsymbol{\phi}}_k^T}{i\omega - \overline{\lambda}_k} \tag{2}$$

or under a general form:

$$\hat{\mathbf{R}}_{\mathbf{xx}}(\omega) = \sum_{k=1}^{N} \frac{\mathbf{A}_k}{i\omega - \lambda_k} + \frac{\mathbf{A}_k^*}{i\omega - \overline{\lambda}_k} \tag{3}$$

where d_k is constant, $\mathbf{A}_k = d_k \boldsymbol{\phi}_k \boldsymbol{\phi}_k^T$ is the residue matrix and $(.)^*$ indicates conjugate and transpose operator.

2.2. Continuous wavelet transform

The Continuous wavelet transform (CWT) of a signal $u(t)$ can be seen as the projection of the signal on a basis function generated from an analyzing function $\psi(.)$ called mother wavelet by dilatation and translation, the shifted and scaled copies of ψ being denoted $\psi_{b,a}$, [21–23]:

$$T_\psi[u](b,a) = \left\langle u(t), \psi_{b,a}(t) \right\rangle, \tag{4}$$

where $\langle \cdot, \cdot \rangle$ is the inner product of $L^2(\mathbb{R})$: $\langle f, g \rangle = \int_{-\infty}^{+\infty} f(t)\overline{g(t)}dt$ and the two parameters, b and $a>0$ are related respectively to time and scale in Eq. (4). According to the normalization chosen for the wavelet, the definition of the wavelets $\psi_{b,a}$ is