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Comments on “Drill-string horizontal dynamics with uncertainty on the frictional force” by T.G. Ritto, M.R. Escalante, Rubens Sampaio, M.B. Rosales [J. Sound Vib. 332 (2013) 145–153]

Zifeng Li

Petroleum Engineering Department, Yanshan University, Qinhuangdao 066004, China

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ABSTRACT

This paper analyzes the mechanical and mathematical models in “Ritto et al. (2013) [1]”. The results are that: (1) the mechanical model is obviously incorrect; (2) the mathematical model is not complete; (3) the differential equation is obviously incorrect; (4) the finite element equation is obviously not discretized from the corresponding mathematical model above, and is obviously incorrect. A mathematical model of dynamics should include the differential equations, the boundary conditions and the initial conditions.

1. Introduction

T.G. Ritto et al. published the original paper on drill string horizontal dynamics [1]. The present paper aims to analyze the correctness and integrities of the mechanical and mathematical models used in their analyses. Similar conditions appears in T.G. Ritto et al. another two papers [2,3].

2. The original statement

This section describes the features of the model employed in the analysis. It should be noticed that this is a first approach to the horizontal drilling dynamics, therefore the model is the simplest possible continuous model. A sketch of the system analyzed is shown in Fig. 1. A constant force is imposed on the left side of the Bottom Hole Assembly (BHA), there is an oscillatory force imposed at the bit, and there is an interaction force between the bit and the rock on the right side of the structure. Besides these forces, gravity and the normal reaction associated to it (that, in the model, are balanced), and friction (the main force in the process) are acting on the drill-string, as shown in figure.

E-mail address: zfli@ysu.edu.cn

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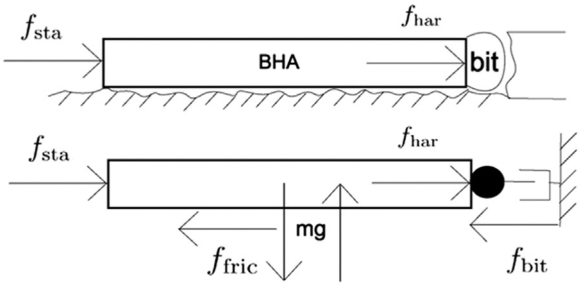

Fig. 1. Sketch of the system analyzed in reference [1].

Only the axial vibrations of the BHA are considered and the equation of motion is given by

$$\rho A\frac{\partial^2 u(x,t)}{\partial t^2}-EA\frac{\partial^2 u(x,t)}{\partial x^2}=f_{\text{sta}}(x,t)+f_{\text{har}}(x,t)+f_{\text{bit}}(\dot{u}(x,t))+f_{\text{fric}}(\dot{u}(x,t))+f_{\text{mass}}(\ddot{u}(x,t)), \tag{1}$$

where u is the axial displacement, ρ is the density of the column, A is the cross sectional area and E is the elasticity modulus. The space and time variables are $x \in [0,L], t \in [0,T]$, where L is the length of the structure and T is the duration of the time analysis. The right-hand side of the equation presents the forces acting on the system, which are depicted in the sequence. A constant force is imposed at $x=0$

$$f_{\text{sta}}(x,t)=F_{\text{sta}}\delta(x), \tag{2}$$

where F_{sta} is the force amplitude, and δ is the Dirac-delta function. This force represents the effect of the drill-pipes on the BHA; it is assumed constant since we are computing the displacement of about 0.1 m of penetration.

A harmonic force is imposed on the system because the driving source of a horizontal drilling is the mud motor, which rotates about a given nominal rotational speed (in a steady operation). Therefore, the axial movement should be excited about the same frequency. The harmonic force f_{har} is given by

$$f_{\text{har}}(x,t)=F_0\ \sin(\omega_f t)\delta(x-L), \tag{3}$$

where F_0 and ω_f are the amplitude and frequency of the harmonic force; this force is applied at $x=L$. The force (per unit length) related to the friction field f_{fric} is given by

$$f_{\text{fric}}(\dot{u}(x,t))=-\mu(x)(\rho A)g\ \text{sgn}(\dot{u}(x,t)), \tag{4}$$

where μ is the friction coefficient field, ρA is the mass per unit length of the structure and g is the gravity. Note that the friction depends on the sign of the speed, which makes it discontinuous. The force related to the bit mass is written as

$$f_{\text{mass}}(\ddot{u}(x,t))=-m_{\text{bit}}\ddot{u}(x,t)\delta(x-L), \tag{5}$$

where m_{bit} is the bit mass located at $x=L$. Finally, we propose a model for the bit–rock interaction force f_{bit}. There is no work in the literature (at the best of the authors' knowledge) that considers the modeling of the bit–rock interaction of a horizontal drilling. In this first attempt to construct a bit–rock interaction model we are assuming that there is a limit force for the bit as the speed of the bit increases. We also assume that the bit force approaches this limit asymptotically (with exponential decay); the model was inspired by the work of Wanhein et al., which was developed in another context. The exponential model is given by

$$\begin{cases} f_{\text{bit}}(\dot{u}(x,t))=[c_1\exp(-c_2\dot{u}(x,t)-c_1]\delta(x-L) & \text{for}\ \ \dot{u}(L,t)>0 \\ f_{\text{bit}}(\dot{u}(x,t))=0 & \text{for}\ \ \dot{u}(L,t)\le 0 \end{cases}, \tag{6}$$

where c_1 and c_2 are the two constants of the bit–rock interaction; this force is applied at $x=L$. The constant c_1 is chosen such that the bit speed is close to a reasonable value (-20 m/h), and c_2 defines the nonlinear shape of the function in the range of analysis.

Using linear interpolation functions for each element that has one axial degree of freedom per node, and assembling the global matrices, the deterministic system of equations are written as

$$M\ddot{\mathbf{u}}(t)+C\dot{\mathbf{u}}(t)+\mathbf{K}\mathbf{u}(t)=\mathbf{f}_{\text{sta}}+\mathbf{f}_{\text{har}}(t)+\mathbf{f}_{\text{bit}}(\dot{\mathbf{u}}(t))+\mathbf{f}_{\text{fric}}(\dot{\mathbf{u}}(t)),\ \mathbf{u}(0)=\mathbf{u}_0, \dot{\mathbf{u}}(0)=\mathbf{v}_0 \tag{7}$$

in which $\mathbf{u}$ is the displacement vector, $\mathbf{M}$ is the mass matrix and $\mathbf{K}$ is the stiffness matrix. The proportional damping matrix $\mathbf{C}=\alpha\mathbf{M}$ (α is a positive constant) is added a posteriori to the computational model. The initial conditions are given by $\mathbf{u}_0$ and $\mathbf{v}_0$. The force vectors $\mathbf{f}_{\text{sta}}, \mathbf{f}_{\text{har}}, \mathbf{f}_{\text{bit}}$ and $\mathbf{f}_{\text{fric}}$ are related to the static, harmonic, bit–rock interaction and frictional forces, respectively. The force related to the bit mass is included in the matrix $\mathbf{M}$, since it depends linearly on the acceleration.