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State feedback control of surge oscillations of two-point mooring system

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ABSTRACT

Stability analysis of surge oscillations of two-point mooring system under state feedback control with time-delay is investigated. The two-point mooring system is harmonically excited and essentially represents a strongly nonlinear Duffing oscillator. In this paper, a frequency domain based method viz. incremental harmonic balance method along with arc-length continuation technique (IHBC) is first employed to identify the primary and higher order subharmonic responses which may be present in such system. The IHBC is then reformulated in a manner to treat two-point mooring system under state feedback control with time-delay and is applied to obtain control of responses in an efficient and systematic way. The stability of uncontrolled responses for primary and higher order subharmonic oscillations is obtained by Floquet's theory using Hsu' scheme; whereas the stability of controlled responses is obtained by applying semi-discretization method for delay differential equation. The study focussed on the controlling primary, higher order subharmonics and chaotic responses by considering appropriate feedback gains and delay by way of (*i*) appreciable reduction of primary, subharmonic responses, (*ii*) exclusion of all higher order subharmonics 2T, 3T, 5T and 9T ($1/n$ subharmonics or period-n solutions), and (iii) reduction of the extent of domain of all instability phenomena represented by various type of bifurcation of solutions, jump phenomena, chaotic responses etc. In the study, negative velocity feedback is observed to be much effective than state feedback for better controlling of surge oscillation of two-point mooring system. Also, the effect of larger gain values is investigated by an extensive parametric study for vibration control with different delay values.

1. Introduction

The complex behaviour of nonlinear system such as periodic, aperiodic and chaotic solutions is generally dealt with active vibration control based on time-delayed feedback control. The delay in the feedback path is expected for several reasons or sometimes introduced intentionally for obtaining desired performance of nonlinear dynamical systems. Generally, the presence of delay either natural or intentional results in the more complex behaviour of the dynamic system. Many researchers [1–4] have investigated the complex dynamics and instability of the controlled systems due to

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unavoidable delays in controllers and actuators.

In the recent past, time-delayed feedback control has been successfully used to control vibrations of various linear and nonlinear systems and stability of such systems is studied by applying various analytical, semi-analytical and numerical methods. Ji and Leung [5] investigated primary, subharmonic and superharmonic resonances of harmonically excited Duffing system with two distinct time-delays. It is shown that appropriate choice of feedback can not only enlarge the critical force amplitude and lessen the peak amplitude of response, it can also eliminate jump and hysteresis phenomena. Udwadia et al. [6–9] have reported the application of time-delayed velocity feedback for controlling the vibrations of structural systems under seismic loading. Atey [10] and Li et al. [11] discussed the use of time-delayed state feedback method in controlling free, forced and parametric vibration of the van der Pol oscillator. Chatterjee and Mahata [12] discussed the efficacy of an active absorber based on the time-delayed displacement difference feedback in controlling friction-driven oscillations. Das and Mallik [13] considered time-delayed PD feedback control of forced vibration of friction-driven systems. Hu et al. [14,15] have reported time-delayed state feedback control of the primary and subharmonic resonances of a forced Duffing oscillator. Saxena et al. [16] investigated the dynamical effects due to integrative time-delay in nonlinear coupled dynamical system. Time-delayed feedback controls of self-excited oscillation have been reported by Chatterjee et al. [17–19]. Popp and Rudolph [20] reported the control vibration due to stick-slip motion. How friction induced self-excited oscillations are affected by high-frequency external excitation is examined by Thomsen [21]. Nonlinear resonance in a damped and periodically driven Duffing oscillator with fixed and integrative time-delayed feedbacks has been studied by Ravichandran et al [22]. Rusinek et al. [23] investigated the influence delay displacement feedback on the primary resonances of classical Duffing oscillator. The influence of the external harmonic excitation on the vibration of the system with time-delay analogous to cutting process control is also studied. In a series of papers [24–27], Maccari presented that proper choices of gain and delay in the feedback scheme can suppress the primary resonance and remove the possibility of existence of two-period quasi-periodic motion. Rusinek et al. [28] studied the presence of stable and unstable periodic orbits in a Duffing oscillator. Then time delayed displacement feedback is employed to destroy the chaotic attractor and to stabilize the periodic orbit. All the works have been performed by asymptotic perturbation method for weakly nonlinear systems with small gains in the feedback path.

From the above study, it is observed that vibration control of a certain class of nonlinear systems are mostly investigated by applying perturbation techniques, numerical integration (NI) techniques, and a straight forward harmonic balance analysis.

The present study is motivated by the need for a better semi-analytical prediction of complex periodic (ultra-subharmonic) via incremental harmonic balance method (IHB), as previous theoretical analysis focused on weakly nonlinear regimes and are mostly suitable for systems with small amplitude of oscillation. Therefore, for dynamics and vibration control analysis of a system having strong nonlinearity with the feedback control law (introduced in the feedback path), a theoretical basis of frequency domain based computer method is necessitated for the study of resonances in systematic way which is robust and better for convergence of solution. To address these issues, an incremental harmonic balance along with continuation technique (IHBC), a systematic computer method [29,30], is applied for time-delayed feedback control of nonlinear surge response behaviour of two-point mooring system under harmonic wave. The two-point mooring system has fairly strong stiffness nonlinearity and is, therefore, expected to show fundamental and subharmonic resonances.

The objective of this paper is to study the amenability of IHBC technique for the analysis of time-delayed feedback control of the strongly nonlinear two-point mooring system and study its efficiency in obtaining suppression of various fundamental and subharmonic resonances present in the system with delayed state feedback. Appreciable reductions in the peak of resonance curves obtained with appropriate choices of the feedback gains and the time-delay from the viewpoint of vibration control are also discussed. The control of complex nonlinear responses such as periodic, aperiodic and chaotic responses present in the strongly nonlinear dynamical system is successfully investigated. Further, an effort has been made to demonstrate the capacity of IHBC method for analysing time-delayed feedback control of such type of system with larger gain values. The effect of larger gain values is investigated by an extensive parametric study for amplitude suppression with different delay values.

In the present study, first, the IHB method along with continuation technique has been developed to study the period-1 frequency response curves for the damped Duffing oscillator (model of a two-point mooring system under surge oscillation) with time-delayed state (displacement and velocity) feedback. Secondly, higher harmonic (i.e. period-2, 3, 5, 9) and chaotic oscillations which are present in the uncontrolled mooring system are traced completely. Finally the efficacy of IHB method in controlling period-1, 2, 3, 5, 9 and chaotic oscillations are investigated with different control parameters. Phase plane plots and time history plots are used for better understanding wherever needed. Results are compared with NI technique at discrete points. The stability of the periodic solution is examined by Floquet's theory using semi-discretization technique. The results show that the primary, subharmonic and chaotic oscillations can be suppressed to the desired level with appropriate choice of delay and gain parameters with broadened stable zone in the responses of the systems.

The paper is arranged as follows: In Section 2, the mathematical model of the two-point mooring system under surge oscillation with time-delayed state feedback is presented. In Section 3, the linear stability analysis of the said mooring system under time-delayed state feedback scheme is investigated for finding the suitable gain and delay regions for control study. In Section 4, the IHB method along with continuation technique has been developed to study the period-1 frequency response curves for the damped Duffing oscillator (model of a two-point mooring system under surge oscillation) with time-delayed state feedback. In Section 5, the stability of delay differential equation (DDE) is discussed applying semi-