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Modeling of guided circumferential SH and Lamb-type waves in open waveguides with semi-analytical finite element and Perfectly Matched Layer method

Paweł J. Matuszyk

Baker Hughes Inc., Houston Technology Center, Houston, TX 77073, USA

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ABSTRACT

The circumferential guided waves (CGW) are of increasing interest for non-destructive inspecting pipes or other cylindrical structures. If such structures are buried underground, these modes can also deliver some valuable information about the surrounding medium or the quality of the contact between the pipe and the embedding medium. Toward this goal, the detailed knowledge of the dispersive characteristics of CGW is required; henceforth, the robust numerical method has to be established, which allows for the extensive study of the propagation of these modes under different loading conditions. Mathematically, this is the problem of the propagation of guided waves in an open waveguide. This problem differs significantly from the corresponding problem of a closed waveguide both in physical and numerical aspect. The paper presents a combination of semi-analytical finite element method with Perfectly Matched Layer technique for a class of coupled acoustics/elasticity problems with application to modeling of CGW. We discuss different aspects of our algorithm and validate the proposed approach against other established methods available in the literature. The presented numerical examples positively verify the robustness of the proposed method.

1. Introduction

Motivation: In this study, we focus on the development of a numerical technique enabling robust and efficient modeling of the circumferential guided waves CGW propagating in pipes embedded into a (multi-layered) structure of infinite radial extent. Due to the dispersive nature of guided waves, their measured amplitudes, velocities, or attenuation can be interpreted in terms of the relevant geometric and physical features of the considered waveguide [1]. Moreover, it is often possible to excite in a structure the selected mode with precisely known physical properties, which can improve the sensitivity of the mode's properties measurement in comparison to other methods and greatly facilitate the interpretation.

This need is driven by the critical requirement of verifying cased well integrity in the oil and gas industry. In order to ensure safe production of hydrocarbons, the quality of the cement (bonding a steel casing to the rock formation) used to maintain zonal isolation and prevent the vertical migration of fluids to the surface must be periodically assessed. This requirement has been made more difficult by the increasing use of the so-called *lightweight cements* (having the acoustic impedance lower than the impedance of well mud). These cements render the industry standard methods using P-wave attenuation with virtually no response difference between that cement or a fluid being present on the backside of the casing. Therefore, a new measurement technique based on the measuring of the propagation of CGW in a steel casing has been recently developed [2].

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This paper presents a combination of Semi-Analytical Finite Element and Perfectly Matched Layer Method methods for modeling the propagation of CGW in open waveguides in the frequency domain. To our knowledge, this is the first attempt to apply this technique to model the propagation of leaky CGW. We also propose a simple algorithm enabling automatic *hp*-discretization of the finite-element mesh that guarantees that the computations are performed in the asymptotic range, *i.e.*, with practically eliminated dispersion errors, and with minimal number of degrees-of-freedom. The specific investigation of the dispersive characteristics of guided circumferential SH and Lamb leaky modes is out the scope of this paper and is left for further studies.

Circumferential guided waves: CGW are the waves propagating in a hollow cylinder in the azimuthal direction in the plane perpendicular to its axis. In general, these waves resemble Rayleigh, Lamb, and transversal (SH) waves in plates; however, they significantly differ (mathematically and physically) from the corresponding plate waves. The study on CGW dates back to the paper by Sezawa [3], where the ansatz $e^{i\nu\theta}$ for the propagation in the circumferential direction was applied to solve the problem of propagation of Rayleigh and Love waves on Earth's surface. In his seminal work [4], Viktorov formulated equations for Rayleigh waves propagating on concave and convex and cylindrical surfaces; he also introduced the concept of angular wavenumber, ν, which is the extension to a real number of an integer circumferential order appearing in the classical solution of the Helmholtz equation in polar coordinates, given by an infinite series. This slight modification has, however, much deeper consequences, as the requirement of the periodicity of the solution in the circumferential direction (and thus the single-valueness of the solution) was dropped off. Formally, this leads to the formulation and solution of the non-periodic Helmholtz problem, which was first introduced by Sommerfeld in the context of the problem of diffraction [5]. Viktorov himself referred without giving any details to the theory of Malyuzhinets being the extension of Sommerfeld method [6]. Later on, this theory was extended to Lamb-type waves [7], and summarized in a book [8]. Meantime, Grace and Goodman introduced complex angular wavenumber to account for the attenuation of CGW propagating in a solid cylinder submerged in water [9]; however, they used in calculations asymptotic forms of Bessel functions to overcome the problem of calculations of complex order Bessel functions. Almost two decades later, the first numerical examples were presented by Qu et al. [10,11] for time-harmonic motion, and by Liu and Qu for the transient case [12]. All these papers investigated Lamb-type waves and introduced a non-dimensional formulation recognizing shape factor (thickness to radius ratio) as the only geometrical factor influencing modes' dispersion, which greatly facilitated the analysis by use of universal families of dispersion curves. Analysis of finite multi-layered cylindrical structures was presented by Valle et al. [13]. A nearly decade later, the first results for much simpler circumferential SH waves were presented [14,15]. The first study on the effect of fluid or solid loading on CGW propagating in pipes was presented by Fong [16,17].

A solution of the problem of propagation of CGW is much more numerically demanding than for corresponding plate modes, especially for multilayered geometry. The classical methods: Transfer Matrix Method [18,19] and Global Matrix Method [20,21] quickly fail due to the necessity of calculation of complicated Bessel functions for large arguments and orders [1,16,22]. Moreover, these methods strongly depend on vulnerable root-searching algorithms in the complex domain [21]. Therefore, another numerical technique, namely semi-analytical finite element method (SAFE), gained importance for solving wave propagation problems in waveguides.

SAFE: In this method, it is assumed that the solution in the direction of a waveguide is analytical (thus, described by smooth analytical functions, *e.g.*, exponentials) and a classical finite element discretization is performed for the cross-section of the waveguide. Hence, a dimensional reduction of the problem is achieved. Additionally, as the solution in the waveguide direction is described analytically, this method is very robust in solving problems for very high frequencies, where the classical approach may fail. In comparison to the traditional approaches, SAFE simplifies handling any irregularities of the problem like anisotropy, intrinsic attenuation, complex geometry of the cross-section, curved axis of the waveguide. The applications of this approach to different vibration problems can be found in [23–31]. The first application of SAFE method to the determination of CGW was presented by Van Velsor [22]; he covered cases of multi-layered closed waveguides and viscoelastic materials.

Open waveguides: There are substantial differences between types of wave modes and numerical treatment of closed and open waveguides. In the former, there exists a finite discrete set of propagating guided modes and an infinite countable set of non-propagating (evanescent) modes, and the energy is conserved, provided all involved media are purely elastic. A countable set of eigenvalues (point spectrum) is a direct consequence of the self-adjointness of the problem. On the contrary, in open waveguides (embedded into an infinite medium) the energy can be radiated into the surrounding medium, which may result in attenuation of wave modes. The excited modes can be classified into three types: trapped, radiation, and leaky modes [32].

From the practical point of view, both trapped and leaky modes are of great importance for ultrasonic NDT. Therefore, in the case of an open waveguide, any numerical method, has to efficiently deal with two major difficulties: unbounded transversal domain and exponential growth of leaky modes. The solution for the former problem is the truncation of the computational domain and application of the appropriate absorbing boundary condition or absorbing layer. If any absorbing layer method is used, the latter problem can be controlled by restricting the computational domain to the close neighborhood of the core of an open waveguide, for which the propagation and attenuation of leaky modes can be calculated. This allows the leaky mode's amplitude not to grow enough to compromise the numerical method.

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