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Characterizing the effective bandwidth of tri-stable energy harvesters

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ABSTRACT

Recently, it has been shown that nonlinear vibratory energy harvesters possessing a tri-stable potential function are capable of harvesting energy efficiently over a wider range of frequencies in comparison to harvesters with a double-well potential function. However, the effect of the design parameters of the harvester on the dynamic response and the effective bandwidth of such devices remains uninvestigated. To fill this void, this paper establishes an analytical approach to characterize the effective frequency bandwidth of harvesters that possess a hexic potential energy function. To achieve this goal, the method of multiple scales is utilized to construct analytical solutions describing the amplitude and stability of the intra- and inter-well dynamics of the harvester. Using these solutions, critical bifurcations in the parameter's space are identified and used to define an effective frequency bandwidth of the harvester. The influence of the electric parameters, namely, the time constant ratio (ratio between the period of the mechanical system and the time constant of the harvesting circuit) and the electromechanical coupling, on the effective frequency bandwidth is analyzed. Experimental studies performed on the harvester are presented to validate some of the theoretical findings.

1. Introduction

Intentional introduction of stiffness nonlinearities into the design of vibratory energy harvesters (VEHs) is a topic which has received significant attention in recent years. Many research studies have shown that, nonlinear VEHs can be used to decrease sensitivity to variations in design and excitation parameters and to enhance performance under random or non-stationary excitations that are typically encountered in realistic environments [1–17].

In general, nonlinear VEHs can be classified into three main categories, namely, mono-, bi- and tri-stable energy harvesters. These harvesters are differentiated based on the shape of their potential energy function. As shown in Fig. 1, mono-stable VEHs are characterized by single well potential function with a global minimum; bi-stable VEHs have a twin-well potential function with two minima (nodes) separated by a local maximum (saddle) also known as the potential barrier; tri-stable VEHs are characterized by a potential energy function that consists of a local minimum at zero with two local minima on either side representing, respectively, the middle and the outer potential wells. The three minima are separated by two local maxima that represent unstable saddles.

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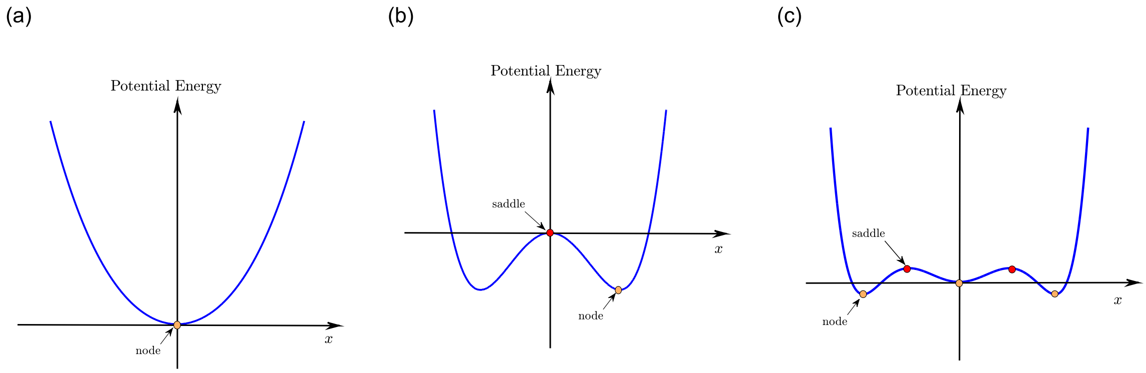

Fig. 1. Schematic of potential energy functions of (a) mono-stable VEH, (b) bi-stable VEH, and (c) tri-stable VEH.

The first two classes of VEHs have been studied extensively in the literature. Nonlinear mono-stable VEHs with piezoelectric and electromagnetic transductions were analyzed in Refs. [4,10,18,19]. These studies demonstrated that the hysteretic (hardening/softening) behavior due to the nonlinearity in these devices can result in large-amplitude responses that extend over a wider range of frequencies as compared to their linear counterparts. However, it was also realized that, depending upon the initial conditions, the harvester is not always guaranteed to operate on the desirable large-amplitude branch of solutions [20]. Moreover, when the nonlinearity and/or the excitation amplitude is small, mono-stable VEHs behave more or less like linear harvesters [21].

To alleviate these issues, many research studies proposed the concept of bi-stability to improve the broadband performance of VEHs. The initial designs of bi-stable VEHs consisted of a piezoelectric cantilever beam with a tip magnet/ferroelectric tip oscillating between two stationary magnets [5,6,10], much like the magneto-elastic structure studied by Moon and Holmes [22]. For a certain separation between the magnets, the harvester exhibits bi-stable characteristics. In another study, Masana and Daqaq [9,19] also proposed a bi-stable harvester which consisted of a clamped–clamped piezoelectric beam subjected to an axial load. When the axial load applied is larger than the critical buckling load of the beam, the harvester becomes bi-stable in nature. Rigorous theoretical and experimental studies were carried out on both these designs of bi-stable VEHs to analyze their response to direct harmonic excitations, random or non-stationary excitations and assess their performance [5–7,9–11,14,16,17,19,23]. A summary of these research findings can be found in Refs. [24,25]. In general, it has been shown that bi-stable VEHs can produce large output voltages over a wide range of frequencies owing to the activation of the large-orbit inter-well oscillations [5,7,9,10,16,17,23]. However, it has also been shown that these large-orbit oscillations cannot be uniquely realized over a wide bandwidth of frequencies and are usually accompanied by chaotic or other undesirable small-amplitude intra-well oscillations. To overcome this issue, Erturk et al. [5] suggested that the piezoelectric layers can be used to create a disturbance that alters the response state (from low-energy to high-energy) potentially by discharging a capacitor thereby improving response bandwidth. More recently, Su et al. [26] considered a cantilever based piezoelectric bi-stable VEH that is similar to those in Refs. [5,6,10], to demonstrate theoretically and experimentally that active tuning of the stiffness and/or damping coefficients via external means such as a motor or electromagnets can be used to ensure that the harvester performs the desirable large-orbit inter-well motions over a wider range of frequencies. However, it was also highlighted that additional circuitry would be necessary to incorporate these active tuning mechanisms into the design of such devices.

Another apparent issue as demonstrated by Panyam et al. [27] is that, depending on the shape of the potential function, there is a threshold value of the excitation level below which it is not possible to activate the desirable inter-well dynamics in a bi-stable VEH. To reduce this threshold, the potential energy function of the harvester can be designed such that it possesses shallow potential wells. Nevertheless, upon reducing the height of the potential barrier, the associated electric output responses of the harvester drop significantly. This poses a challenge in designing these devices, especially when the magnitude and the nature of excitation are unknown.

In light of such challenges, recent research has focused on exploring newer designs of nonlinear VEHs that are capable of producing sustained large-amplitude electric responses. In one demonstration, Zhou et al. [28–30] consider a tri-stable piezoelectric energy harvester which consists of a cantilever beam with a tip magnet oscillating between two stationary magnets much like the earlier designs of the bi-stable VEHs adopted in Refs. [5,6,10,26,31]. They showed that, for a certain orientation and separation distance between the two stationary magnets, the harvester exhibits a tri-stable potential function (as shown in Fig. 1(c)). Preliminary experiments and numerical simulations presented in their work illustrated that the tri-stable harvester produces higher voltage outputs as compared to the bi-stable one over a wider frequency range even for lower-amplitude base excitations. However, their study was limited to numerical and experimental investigations and did not include the effect of design parameters on the response of such devices. In more recent studies Jung et al. [32] and Zhou et al. [33] use a tri-stable harvester very similar in design to one adopted in Refs. [28,29] to analytically and experimentally study its response characteristics. The study by Jung et al. [32] specifically evaluates the conditions (inclination angle of the stationary magnets) under which the harvester exhibits mono-, bi-, and tri-stable characteristics. Furthermore,