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Bifurcation analysis of coupled lateral/torsional vibrations of rotor systems

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ABSTRACT

This paper presents a numerical method to analyze the bifurcation of coupled lateral/torsional vibrations of rotor systems. Based on a Hamiltonian approach, a three degree-of-freedom dynamic model of a rotor is derived. Nonlinear ordinary differential equations are derived from the dynamic model. The stability of the equilibrium and linear normal modes (LNMs) are analyzed using a linearized matrix of the system equation. For bifurcation analysis of the periodic orbits, a nonlinear normal modes (NNMs) computation algorithm is performed using multiple shooting methods and pseudo-arclength continuation. Multiple shooting points are continued from LNMs near equilibrium, bifurcation points of the NNMs are detected from the stability change of the periodic orbits during the continuation. The proposed stability analysis, an NNMs computation of coupled lateral/torsional vibration, is demonstrated using two different rotor models: a system with strong eccentricity, and a system with weak eccentricity.

1. Introduction

Rotor vibration is a complicated dynamical phenomenon, which can cause serious problems. For typical rotor vibration analysis, lateral and torsional vibrations are usually analyzed separately. However, in realistic rotor systems, lateral/torsional vibration occurs as a coupled motion. For lateral/torsional vibration coupling, many researchers have completed theoretical works. Mohiuddin and Khulief [1] derived an elasto-dynamic model by using the Lagrangian approach. Gosiewski [2] derived a three-dimensional rotor dynamic model under unbalance that accounts for two lateral displacements and one torsional angle and obtained system stability by using the Evans method from the control theory. Al-Bedoor [3] presented a model for the coupled torsional and lateral vibrations of unbalanced rotors that accounts for rotor to stator rubbing. The system equation was derived from the Lagrangian, nonlinear equations and was analyzed numerically by using a predictive-corrective time integration algorithm. Li [4] developed a nonlinear, coupled lateral/torsional vibration model of a rotor-bearing-gear coupling system based on engagement conditions of gear couplings. Numerical analysis was performed to calculate time solutions. Hsieh [5] developed a modified transfer matrix method for analyzing coupled lateral/torsional vibrations of a symmetric rotor-bearing system with an external torque. Yuan [6] developed a full-degree-of-freedom dynamic model of a Jeffcott rotor by using the Lagrangian approach. The harmonic balance method and Folquet theory are combined to analyze the stability of the system equations, where the frequencies under analysis are harmonics of the rotating speed. As the coupled lateral/torsional vibration system is nonlinear, bifurcation analysis of equilibrium and

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Nomenclature

b_i	i-th velocity vector in multiple shooting method
$\mathbf{dv}_{(j)}$	Direction vector of j-th predictor step
$\bar{e}$	Mass eccentricity
$\bar{\mathbf{F}}$	Derived system ODE
$\bar{\mathbf{F}}_{\xi}$	Jacobian of $\bar{\mathbf{F}}$
H	Hamiltonian in fixed coordinates
$\hat{H}$	Hamiltonian in rotating coordinates
I	Moment of inertia
K_{xx} , K_{yy}	Lateral stiffness
K_t	Torsional stiffness
m	Mass of rotating disk
$\mathbf{M}$	Monodromy matrix
$\mathbf{M}_i$	i-th part of monodromy matrix in multiple shooting method
$\mathbf{p}$	Conjugate momenta in rotating coordinates
p	Phase condition
$\mathbf{q}$	Displacement vector in rotating coordinates
$\mathbf{r}$	Conjugate momenta in fixed coordinates
r_{mr}	Maximum real eigenvalue of $\bar{\mathbf{F}}$
$\mathbf{s}$	Displacement vector in fixed coordinates
s	Partition of unit interval during the multiple shooting method
$\mathbf{S}$	Shooting function
$\bar{\mathbf{S}}$	Multiple shooting function
$\mathbf{v}_l,\bar{\mathbf{v}}_l$	Eigenvectors of l-th mode
$x-y$	Lateral displacements of fixed coordinate
$\mathbf{z}$	Phase space representation of fixed coordinates
Δs_i	i-th partition size during the multiple shooting method
θ	Torsional deflection angle
θ_{lp}	Phase angle between ξ_1 and ξ_p at l-th mode
σ_{ω_l}	Krein signature of l-th mode
ω_0	lateral natural frequency
ϕ	Rotation angle
λ_m	Maximum non-trivial Floquet multiplier
$\boldsymbol{\xi}$	Phase space representation of rotating coordinates
$\boldsymbol{\xi}_e$	equilibrium of the system
ψ_{lp}	Phase of the p-th displacement of l-th mode

Sub/superscript

i	Multiple shooting point number
j	NNMs continuation number
k	Iteration index during the corrector step
l	mode number
p	displacement number

periodic solutions is essential for a deep understanding of the system. However, as far as the author knows, there is no literature describing theoretical approaches for the bifurcation analysis of lateral/torsional vibrations. The aim of this study is to apply bifurcation analysis to rotor dynamics by analyzing the stability of equilibrium, periodic orbits. Stability of the equilibrium can be easily analyzed using traditional linearization techniques. In contrast, analyzing periodic orbits of nonlinear dynamic systems is much more complicated.

Periodic orbits of nonlinear dynamic systems can be analyzed by nonlinear normal modes (NNMs), which give theoretical and mathematical tools for various nonlinear systems. The theory and applications of NNMs were developed by various researchers. NNMs were first proposed by Rosenberg [7], who defined NNMs as synchronous oscillations in nonlinear systems. Rand [8] presented a direct method for locating normal modes in certain conservative nonlinear two degree of freedom dynamical systems. Vakakis [9] examined free oscillations of a strongly nonlinear discrete oscillator by computing non-similar NNMs, in which motions are represented by curves in the configuration space of the system. Shaw and Pierre [10–12] derived NNMs by using an invariant manifold approach and analyzed nonlinear discrete and continuous systems by using this approach. Peeters [13] defined NNMs as a periodic motion in nonlinear dynamic systems and developed numerical iteration continuation techniques for NNMs.

In this research, bifurcation analysis of a equilibrium is performed by the linearization of a nonlinear system equation, and bifurcation analysis of a periodic orbit is performed by a continuation technique for NNMs. Another objective of this study is to show that unstable self-excited rotating speed regions can exist, which was not pointed out in previous studies of coupled lateral/torsional vibrations.

This paper is organized as follows. In the second section, a nonlinear nonautonomous Hamiltonian is derived from the Lagrangian. Then, autonomous nonlinear ordinary differential equation (ODE) is derived by using symplectic transformation. In the third section, a method to analyze the bifurcation of a equilibrium and linear normal modes (LNMs) of the system is obtained. In the fourth section, numerical algorithms for calculating the NNMs and methods for stability analysis of NNMs are introduced. The bifurcation analyses of equilibrium, periodic solutions are then applied to two different rotor models.

2. Dynamic model

2.1. Assumptions and definitions

The schematic diagram of a system dynamic model, which is supported by rigid bearings, is shown in Fig. 1. This model adopts Jeffcott's model, which consists of a rigid disc and a massless flexible shaft. Derivation of the equation of motion is