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An integral formulation for wave propagation on weakly non-uniform potential flows

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ABSTRACT

An integral formulation for acoustic radiation in moving flows is presented. It is based on a potential formulation for acoustic radiation on weakly non-uniform subsonic mean flows. This work is motivated by the absence of suitable kernels for wave propagation on non-uniform flow. The integral solution is formulated using a Green's function obtained by combining the Taylor and Lorentz transformations. Although most conventional approaches based on either transform solve the Helmholtz problem in a transformed domain, the current Green's function and associated integral equation are derived in the physical space. A dimensional error analysis is developed to identify the limitations of the current formulation. Numerical applications are performed to assess the accuracy of the integral solution. It is tested as a means of extrapolating a numerical solution available on the outer boundary of a domain to the far field, and as a means of solving scattering problems by rigid surfaces in non-uniform flows. The results show that the error associated with the physical model deteriorates with increasing frequency and mean flow Mach number. However, the error is generated only in the domain where mean flow non-uniformities are significant and is constant in regions where the flow is uniform.

1. Introduction

Predicting noise radiation from complex sources in moving flows is relevant to the automotive, energy and aeronautical industries. Noise radiation from turbofan nacelles and from other aircraft sources is a problem of particular interest in the aviation sector. Numerical simulation of noise radiation and scattering can significantly reduce costs for design and certification. However, an efficient numerical method for high frequency noise propagation on non-uniform moving flows has not yet been demonstrated. Solving high-frequency short-wavelength problems on moving flows remains computationally expensive. In the aeronautical industry, noise propagation on non-uniform flows is typically predicted using finite element methods (FEM) [1], discontinuous Galerkin methods (DGM) [2] and high order finite difference schemes [3].

Although volume based methods, such as FEM, DGM and finite difference schemes are able to solve wave propagation on a non-uniform flow, predicting noise radiation in unbounded domain requires the computational domain to be truncated. The truncation of the domain allows acoustic waves to be damped in a non-physical absorbing zone [3–5] and satisfy the

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radiation condition at the outer boundary of the domain. Moreover, these methods suffer of dispersion error and pollution effects [6]. These features are relevant limitations in case of noise radiation for large-scale short-wavelength problems.

On the other hand, numerical methods based on boundary integral formulations, such as the boundary element method (BEM) [7], inherently satisfy the radiation condition in the kernel and allow wave propagation in unbounded domains to be solved more effectively than in the case of volume based methods. Moreover, the fast multipole BEM (FMBEM) is an efficient algorithm to solve wave radiation and scattering for large-scale short-wavelength problems [8]. However, BEM can only solve wave propagation exactly on uniform mean flows. Extending this method to non-uniform flow regions would be beneficial to a number of applications, such as forward fan noise acoustic installation effects. A surface integral formulation including non-uniform flow effects would also extend the applicability of wave extrapolation methods. These approaches use an integral formulation defined on a closed surface on which the acoustic field is sampled from an 'inner' domain to radiate the solution to the far field. At the moment these methods are limited to uniform flow [9,10].

Current boundary element modelling practices use the Lorentz transformation [11,12] to solve wave propagation on a uniform mean flow. This variable transformation allows the uniform flow Helmholtz equation to be reduced to the standard Helmholtz problem without approximations. By means of a Lorentz transformation, BEM solvers for the standard Helmholtz equation can therefore be used for wave propagation on uniform flows. However, due to the variable transformation, the physical space is deformed in the direction of the mean flow. The deformation of the domain complicates the formulation of the boundary conditions and the implementation of the transmission conditions for coupled formulations [13,14]. Alternatively, this drawback can be overcome by using an integral formulation in the physical space as proposed by Wu and Lee [15] in the frequency domain and by Hu [16] in the time domain.

For BEM, only approximate formulations are available for representing non-uniform mean flow effects. Astley and Bain [17] provided an approximate formulation for wave propagation on low Mach number mean flows based on Taylor's transformation [18,19]. In the same work, Astley and Bain [17] reported an error analysis for Taylor's wave equation showing that the accuracy of the physical model depends only upon the mean flow Mach number and the characteristic length scales of the acoustic waves and the mean flow. On the other hand, Tinetti and Dunn [20] provided a generalized local Lorentz transformation to represent the effect of non-uniform mean flows on wave propagation. However, the method has been restricted to mean flow fields with small gradients. Another approach is to move the terms including non-uniform flow effects to the right hand side of the equation and treat them as sources in the domain. The dual-reciprocity method (DRM) [21] is then used to convert the domain integrals into boundary integrals. The absence of a robust method to define interpolating source functions for the DRM restricts the applicability of this approach. Thereby, modeling non-uniform flow effects for BEM is still an open problem.

In this article, we present, in the physical space, an integral formulation with non-uniform flow based on a combination of the physical models associated with the Taylor and Lorentz transformations. The proposed physical model is an approximate formulation of the full linearized potential wave equation for isentropic compressible flows. The integral formulation derived applies to sound radiation on a *weakly non-uniform* potential mean flow. Consider the mean flow as a sum of a uniform and a non-uniform component which vanishes at infinity. The term *weakly non-uniform* indicates that the non-uniform portion of the mean flow is small compared to the uniform part. A free field Green's function is also determined for a subsonic weakly non-uniform flow as a kernel for the integral equation. Moreover, an error analysis is presented to extend and to revisit the estimate provided by Astley and Bain [17]. This analysis shows the dependency of the error related to the physical model on the mean flow Mach number and on the frequency. The proposed formulation will be shown to improve the accuracy of the model compared to existing formulations based on either Taylor or Lorentz transform being applied separately.

The paper is structured as follows. Section 2 presents the physical models. In Section 3 integral formulations are derived for wave propagation on weakly non-uniform mean flows and the Taylor formulation in the physical space. The Green's functions associated with the integral formulations are derived in Section 4. Boundary element formulations consistent with the proposed integral solutions are presented in Section 5 for an arbitrary source distribution. In Section 6, a dimensional error analysis is developed to describe the limitation of the proposed solutions. Finally, in Section 7 some numerical results are presented to benchmark the integral formulations.

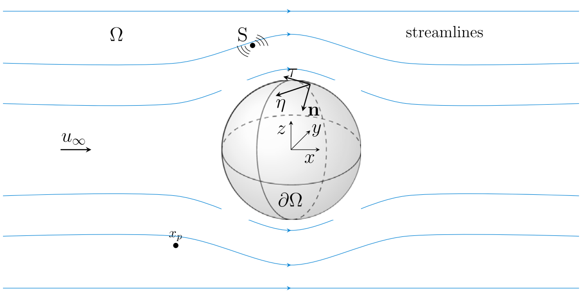

Fig. 1. Schematic diagram of the reference problem showing mean flow streamlines in the solution domain Ω.