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Estimation of the transient response of a tuned, fractionally damped elastomeric isolator

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ABSTRACT

This article addresses the frequency dependent properties of elastomeric vibration isolators in the context of lumped parameter models with fractional damping elements. A mass is placed between two fractional calculus Kelvin-Voigt elements to develop a minimal order system for the example case of a conventional elastomeric bushing typical of automotive suspension systems. Model parameters are acquired from measured dynamic stiffness spectra and a finite element model. The minimal order system model accurately predicts dynamic stiffness in both broadband resonant behavior as well as the lower-frequency regime that is controlled by damping. For transient response analysis, an inverse Laplace transform of the dynamic stiffness spectrum is taken via the Residue Theorem. Since the fractional calculus based solution is given in terms of problematic integrals, a new time-frequency domain estimation technique is proposed which approximates time-domain responses for a class of transient excitation functions. The approximation error is quantified and found to be reasonably small, and tractable closed-form transient response functions are provided along with a discussion of numerical issues.

1. Introduction

Elastomeric isolators, mounts, and bushings are widely used in machines, vehicles, and buildings. In automotive suspension, for instance, bushings are used extensively to ensure vibration isolation and impedance mismatch between critical subsystems. Such isolators have been analytically studied with continuous system theory [1–3] as well as lumped parameter models on both the component [4–6] and system [7] level, often with many simplifications. In many cases, isolators exhibit nonlinear behavior [3–5,8] which may be more readily apparent from transient responses [4,5]. In recent decades, increased attention has been given to the use of fractional order derivatives to describe the viscoelastic behavior of elastomeric materials [9–14]. Fractional calculus theory has many physical applications [9,10] and has been applied to viscoelasticity in the context of pure constitutive relations [10–13], material models suitable for finite element analysis [12], and lumped system modeling [14,15]. Despite the many approaches to using fractional calculus for viscoelasticity [10], it is still not widely used in practice, presumably due to the difficulty of the mathematical sophistication which is often required to produce useful results. Prior approaches to employ fractional calculus can be divided into two categories: analytical

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transformations of fractional calculus formulations based on Cauchy integrals [9–11,13,14] and those numerical estimation of fractional derivatives based on the variations of the Grünwald definition [9,10,12,15]. Each methodology presents unique strengths and weaknesses, but this article aims to adopt and build on the analytical approach by introducing an estimation procedure which may simplify the difficult time-domain transformation calculations [13,14], extending the method to a new class of system. The chief goal is to develop semi-analytical, time-domain response estimates to a class of transient excitation signals when applied to a tuned elastomeric isolator (with inertial effects) in the context of a minimal order model.

2. Problem formulation

Elastomeric isolation components used in automotive suspension exhibit significant frequency-dependent stiffness behavior over a wide range of frequencies. Such "tuned" properties result from inertial and damping effects in the material, each being dominant in a different frequency regime. Capturing the relevant physics in a reduced-order, lumped-parameter model which is effective over a broad frequency range is challenging. Recently, Noll, et al. developed a lumped parameter model for an elastomeric joint which clarified the frequency-dependent stiffness by capturing internal mass effects [6]. Their model offers good broadband dynamic stiffness predictions, but significant error is found in the lower-frequency regime (say, up to 100 Hz). This error is linked to the damping mechanism assumed in the model [6]. Neither structural nor viscous damping is able to produce the low-frequency behavior with a minimal-order model [15], although large, empirical viscoelastic networks may reproduce the effect on an *ad hoc* basis. A damping mechanism based on fractional calculus is expected to yield superior predictions as assumed in this article.

The objectives of this article are as follows: 1. Develop a uniaxial minimal order model which can effectively simulate the dynamic stiffness of a production bushing, capturing both low-frequency and broadband behavior covering the first resonance (0–1 kHz), and 2. Propose a new estimation technique to yield time-domain solution approximations for transient excitation using fractional calculus. Spectral characterization of such viscoelastic isolators is useful but insufficient, as the assumption of purely harmonic excitation is unrealistic and may mask properties of the elastomeric system that are relevant under aperiodic or transient excitation. Studying transient responses requires a time-domain representation of the system, such as an impulse response function. Fractional dampers preclude conventional inverse Laplace transform calculations to obtain the impulse response; however, the transform may be obtained using the Residue Theorem and multi-domain estimation techniques. The step response is useful in the analysis of elastomeric isolators since it includes an abrupt shift in the operating loads which may excite interesting behavior from any amplitude-dependent nonlinearities in the system [3,5,10,12,15]. Determination of such amplitude-dependent behavior is beyond the scope of this work; nevertheless, a "step-like" input will be used as an example for a realistic transient excitation. Additional complexities may be observed in elastomeric materials such as temperature dependence, aging effects, and anisotropy [16]. Since dynamic behavior of viscoelastic materials often exhibit limited sensitivity to these effects for small perturbations about an operating point, these are beyond the scope of this work.

3. Spectral characterization and minimal order model

Elastomeric isolators are typically characterized in terms of dynamic stiffness spectra [1–3,6,7]. A comparison between the measured cross-point stiffness magnitude spectrum and finite element predictions of a laboratory bushing (very similar to production devices [6]) is given in Fig. 1. Details and parameters of the finite element model are already reported in [6]. Good broadband accuracy is achieved by the finite element model; however, at low frequencies (below 100 Hz), the measurement reveals a damping mechanism which is not incorporated in the model. Conversely, the measured dynamic stiffness spectrum only goes up to 600 Hz (due to the limitations of the dynamic elastomer test machine), whereas finite element models can extend to larger bandwidths to capture stiffness peaks due to the internal mass. Fig. 2 depicts the simulated dynamic stiffness spectra for a production bushing [6], including both driving-point and cross-point stiffness

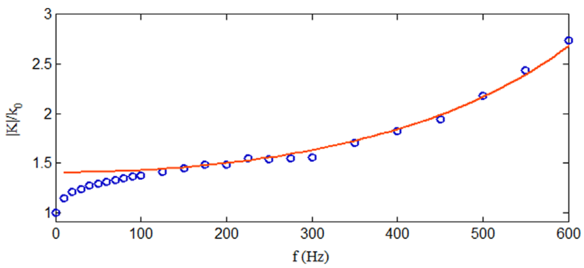

Fig. 1. Dynamic stiffness predictions of an elastomeric isolator showing insufficiency of structural damping mechanism available in finite element model [6]. Key: ○ - Measurement; ▬ - Finite element model with structural damping.