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A quasi-modal parameter based system identification procedure with non-proportional hysteretic damping

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ABSTRACT

This paper introduced a modal parameter based identification procedure to identify the equivalent system of structures under harmonic excitations. The developed identification technique assumed non-proportional hysteretic damping in the equivalent system, which would be applicable in identifying more general structures. By introducing quasi-modal parameter, modal analysis equation was decoupled under physical coordinate; hence, the modal parameters of each vibration mode are identified independently. Double iteration algorithm was developed to solve the derived non-linear identification equation with complex unknowns. The developed identification procedure was applied to identify the equivalent system of a numerical model in order to evaluate the feasibility of the technique in practice. The identification procedure was also applied to identify an experimental mass and bar rig for validation purpose. Identification results showed that the identification procedure could identify accurately and robustly the equivalent system with non-proportional hysteretic damping assumption; hence, it is likely to be applicable in the field.

1. Introduction

The overall dynamic behaviour of rotating machinery is significantly affected by its supporting structure, or foundation. Therefore, the equivalent system of rotor foundation is necessarily identified in the recent model based research on rotor dynamics [1–4]. A promising approach for such system identification uses the vibration measurement of the foundation with the rotating machinery in operation [5], to identify the relevant modal parameters for an equivalent system, defined as a system which, when substituted for the actual foundation, reproduces the vibration behaviour over the operating speed range of interest. If successful, such an identification technique would be applicable to identify the supporting structures of existing rotating machinery installations, using readily available monitoring instrumentation.

In general, modal parameter identification procedure solves the modal analysis equation to identify the modal parameters of all vibration modes simultaneously, which include natural frequencies, mode shapes, modal masses and damping ratios/factors [6–8]. Serving as the measurement data input are the harmonic excitation forces and displacement responses on the foundation, caused by the existing rotor unbalance when the machinery is in operation. Under laboratory environment, the displacement responses of the foundation at any given rotor operating speed can be measured by

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Nomenclature

$\mathbf{A}^*$; a^*_{jk}	complex quasi-modal matrix $=(\mathbf{\Phi}^{*-1})^{\mathrm{T}}$; elements of $\mathbf{A}^*$
$\tilde{\mathbf{f}}$;$\tilde{\mathbf{F}}$	excitation force vector; amplitude thereof
$\overline{F}_j$; $\overline{\overline{F}}_j$	excitation force element $=F_j^R$ or F_j^I; excitation force element $=F_j^I$ or $-F_j^R$
$\mathbf{C}$; $\mathbf{H}$	non-proportional hysteretic damping matrix; $\mathbf{H}=\Omega\mathbf{C}$
$\mathbf{I}$	identity matrix
$\mathbf{k}^*$	complex modal stiffness matrix
$\mathbf{K}^*$	complex stiffness matrix $=\mathbf{K}+i\mathbf{H}$
$\mathbf{K}$	system stiffness matrix
$\mathbf{m}^*$	complex modal mass matrix
m_k^*	complex modal mass of the kth mode
$\overline{m}_k^*$	inverse of complex modal mass of the kth mode$=\overline{m}_k^*=(m_k^*)^{-1}$ elements
$\mathbf{M}$	system mass matrix
$\tilde{\mathbf{Q}}$	amplitude of modal displacement vector
$\tilde{\mathbf{x}}$; $\tilde{\mathbf{X}}$	displacement vector; amplitude thereof
$\overline{X}_j$; $\overline{\overline{X}}_j$	displacement element $=X_j^R$ or X_j^I; displacement element$=X_j^I$ or $-X_j^R$
η_k	hysteretic damping loss factor of the kth mode
$\boldsymbol{\lambda}^*$	complex eigenvalue matrix $=\mathbf{m}^{*-1}\mathbf{k}^*$
λ_k^*	k^{th} complex eigenvalue $=\omega_k^2+i\omega_k^2\eta_k$
$\mathbf{\Phi}^*$; Φ^*_{jk}	complex modal matrix; elements of $\mathbf{\Phi}^*$
φ^*_{jk}	combined elements;
Ω	excitation frequency
ω_k	damped natural frequency of the kth mode
$\blacksquare^R$; $\blacksquare^I$	real part of $\blacksquare$; imaginary part of $\blacksquare$

accelerometers; however the foundation excitation forces need to be determined indirectly. Nevertheless, a few different techniques to derive the excitation forces have been experimentally validated by different researchers [9–11], providing different levels of accuracy in force estimation.

In modal approach, proportional damping model is commonly adopted since it is able to reduce the complexity of the non-linear modal analysis equation and provide general approximation [12,13]. Hence, a few modal parameter identification techniques with proportional damping assumption have been developed, enabling acceptable identification results in experimental validation [14–16]. However, in real application, the structures to be identified do not necessarily have proportional damping and may even involve nonlinearity [17]. As a result, in the recent years, a more accurate assumption of non-proportional damping is preferred when modelling large and complex structures in the industries [18–20]. Hence, to develop the technique for equivalent system identification under non-proportional damping assumption is in order.

In our previous research, we developed a quasi-modal parameter based identification technique to identify the equivalent system of rotor foundation under harmonic excitation, under proportional damping assumption. The developed technique decoupled the modal analysis equation under physical coordinate and solved for the modal parameters of each vibration mode independently [21]. In this paper, the quasi-modal parameter based identification technique will be further extended to identify the equivalent system under non-proportional hysteretic damping assumption. As a first step, the developed technique will be applied to identify the equivalent system of a 5 degree of freedom (DOF) numerical system under harmonic excitation forces. The numerical 'measurement' data with different noise level will be used to evaluate whether the developed technique is applicable in practice. After that, the equivalent system of an experimental mass and bar rig will also be identified to assess the capability of the technique to cater for practical instrumentation errors, instrumentation limitations as well as to assess data processing capabilities. The experimental evaluation of the identification technique for application to as complex a system as rotating machinery is outside the scope of this paper and left for future work.

2. Identification theory

An n DOF equivalent system with hysteretic damping, which is expected to accurately represent the original structure, has the equation of motion [22]:

$$\mathbf{M}\ddot{\tilde{\mathbf{x}}}+\mathbf{C}\dot{\tilde{\mathbf{x}}}+\mathbf{K}\tilde{\mathbf{x}}=\tilde{\mathbf{f}}. \tag{1}$$

Here, $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are assumed to be the symmetrical mass, damping and stiffness matrices of the equivalent system respectively. The hysteretic damping matrix is further represented as $\mathbf{C}=\mathbf{H}/\Omega$ since it is defined to be frequency dependent [22]. The elements in vector $\tilde{\mathbf{x}}$ are the n independent displacements chosen to coincide with the convenient measurement locations on the original structure, which include the application points of excitation forces. The elements of vector $\tilde{\mathbf{f}}$ are the excitation forces acting at those selected locations. Assuming the structure is under harmonic excitation with frequency equals Ω, one can have the representations $\tilde{\mathbf{f}}=\tilde{\mathbf{F}}e^{i\Omega t}$ and $\tilde{\mathbf{x}}=\tilde{\mathbf{X}}e^{i\Omega t}$; therefore Eq. (1) is written as

$$-\Omega^2\mathbf{M}\tilde{\mathbf{X}}+\mathrm{i}\mathbf{H}\tilde{\mathbf{X}}+\mathbf{K}\tilde{\mathbf{X}}=\tilde{\mathbf{F}}. \tag{2}$$

Defining complex stiffness $\mathbf{K}^*=\mathbf{K}+\mathrm{i}\mathbf{H}$, Eq. (2) is written as

$$-\Omega^2\mathbf{M}\tilde{\mathbf{X}}+\mathbf{K}^*\tilde{\mathbf{X}}=\tilde{\mathbf{F}}. \tag{3}$$