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Theoretical and numerical modeling of membrane and bending elastic wave propagation in honeycomb thin layers and sandwiches

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ABSTRACT

Elastic wave propagation in honeycomb thin layers and sandwiches is investigated theoretically and numerically by using the Bloch wave transform, so the modeling of a unique primitive cell is sufficient to understand the wave propagation phenomena through the whole periodic structure. Both in-plane (with respect to the plane of the honeycomb layer) and out-of-plane waves are analyzed by developing finite element models formulated within the framework of the Mindlin–Reissner theory of plates. The dispersion relations and the phase and group velocities as function of frequency and of direction of propagation are calculated. The anisotropic behaviors and the dispersive characteristics of the studied periodic media with respect to the wave propagation are then analyzed. According to our numerical investigation, it is believed that the existence of bandgaps is probably not possible in the frequency domain considered in the present work. However, as an important and original result, the existence of the “backward-propagating” frequency bands, within which Bloch wave modes propagate backwards with a negative group velocity, is highlighted. As another important result, the comparison is made between the first Bloch wave modes and the membrane and bending/transverse shear wave modes of the classical equivalent homogenized orthotropic plate model of the honeycomb media. A good comparison is obtained for honeycomb thin layers while a more important difference is observed in the case of honeycomb sandwiches, for which the pertinence of finite element models is discussed. Finally, the important role played by the honeycomb core in the flexural dynamic behaviors of the honeycomb sandwiches is confirmed.

1. Introduction

Honeycomb sandwiches are widely used in the aerospace and automotive industry due to their high strength-to-weight ratio. Indeed, they are made of a lightweight honeycomb layer sandwiched between two thin, stiff and high-strength facing skins, commonly made of fiber-reinforced laminates. The honeycomb core, in spite of its small additional weight, plays a key role in the bending and transverse shearing strength of the sandwiches. While the classical homogenized models, defined for example by Gibson et al. [1,2], offer an efficient and reliable solution for investigating the static or low frequency

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dynamic behaviors of honeycomb thin layers and sandwiches, they fail to give appropriate simulation results in the so-called high frequency domains, in which the involved wavelengths are close to or shorter than the characteristic lengths of the honeycomb cellular structure. Unfortunately, these high frequency domains can be low enough, particularly for the bending/transverse shear waves, to fall into the frequency ranges of interest of many industrial applications [3,4]. Therefore relevant modeling, which properly accounts for interactions between the honeycomb cellular microstructure and the propagation of high frequency elastic waves, is of great interest.

As indicated in our previous works on the two-dimensional periodic beam lattices, we have developed analytical and numerical tools based on the Bloch wave theory to investigate high frequency elastic wave phenomena in cellular periodic structures [5,6]. Initially employed in quantum mechanics and photonics crystal [8], the Bloch wave theory has been progressively applied in periodic mechanical structures. The basic idea of the Bloch wave theory is the decomposition of any nonperiodic function defined on a periodic structure upon a basis of periodic functions, called Bloch wave modes [5–11,24,25]. Therefore by investigating the Bloch wave modes in a unique primitive cell, the wave propagation phenomena through the whole periodic structure can be understood and modeled, saving lots of analyses and simulation efforts. Dual to the primitive cell of periodicity, another basic concept of the Bloch wave theory is the so-called first Brillouin zone [12]. Indeed, it can be shown that the wave vector $\mathbf{k}$ of the elastic wave propagation in a periodic structure is also periodic in the phase space, also known as the reciprocal space, and the single periodic reciprocal cell is the first Brillouin zone. Thus, the full dispersion relation $\omega(\mathbf{k})$, expressing the angular frequency ω as a function of the wave vector $\mathbf{k}$, can be obtained by only evaluating it over the first Brillouin zone.

Since the 1970s, much work on how harmonic waves propagate through networks with multiple coupling between adjacent periodic elements has been carried out. The dispersive characteristics of the periodic structures have been highlighted, especially the existence of some frequency intervals, within which waves cannot propagate across two adjacent cells and energy exchanges are stopped [7,13]. Such frequency intervals form frequency bandgaps that represent one of the most important and interesting characteristics of the periodic structures. Otherwise with respect to the wave propagation, it was found that periodic structures display anisotropic behaviors characterized by preferential directions of propagation and energy flow. Both the anisotropic behaviors and the dispersive characteristics of periodic structures obviously depend on the impedance mismatch generated by periodic mechanical and geometric properties within one cell or between adjacent cells [14–21]. Hence periodic cellular structures can be considered as frequency or/and spatial filters depending on different pattern and network designs. Therefore more and more attention has been paid to microstructural optimization for well-designed filters [22,26,29].

In the present work, special emphasis is put on investigating the influence of mechanical/geometric parameters of the honeycomb thin layer and sandwiches on the bandgaps, the anisotropic behaviors and the dispersive characteristics with respect to the elastic wave propagation and energy flow exchange. The search of the frequency bandgaps within the frequency domain considered in the present work is performed and extended to "backward-propagating" frequency bands, within which Bloch wave modes propagate backwards with a negative group velocity. Indeed, such "backward-propagating" frequency bands have already been found in the two-dimensional periodic beam lattices studied in our previous works [6]. Furthermore by taking the two-dimensional periodic beam lattices as a reference, the relevance of our numerical models using Mindlin–Reissner plate elements is analyzed and the role played by the honeycomb core in the flexural dynamic behaviors of the honeycomb sandwiches is discussed. Finally, a comparison is made between the first Bloch wave modes and the membrane and bending/transverse shear wave modes of the classical equivalent homogenized orthotropic plate model of the honeycomb media.

The remainder of this paper is organized as follows: A brief presentation of the Bloch wave theory is given in Section 2. In Section 3, the Bloch eigenproblems are defined for the hexagonal honeycomb thin layers and sandwiches. Then numerical analyses of the elastic wave propagation in the honeycomb thin layers and sandwiches are presented in Section 4. Finally the main conclusions are given in Section 5.

2. Bloch wave theory applied to the elastic wave analysis in periodic structures

We give a brief presentation of Bloch wave theory applied to the analysis of elastic wave propagation in periodic structures.

2.1. Geometry parameterization and notations

Consider a d-dimensional periodic structure Ω with primitive cell Q_0 ($d=1,2$ or 3). Any point $\mathbf{p}$ of Ω can be parametrized with respect to a point $\mathbf{q}$ of Q_0 in the following way:

$$\mathbf{p} = \mathbf{q} + n_j \mathbf{g}_j \tag{1}$$

with $\{n_j\}_{j=1,\cdots,d} \in Z^d$ and $\{\mathbf{g}_j\}_{j=1,\cdots,d}$ the basis of periodicity vectors of Ω. The basis $\{\mathbf{g}_j\}_{j=1,\cdots,d}$ is generally not orthonormal and its choice not unique. In the whole paper, the Einstein summation convention is used: $n_j\mathbf{g}_j = \sum_{j=1}^{d} n_j\mathbf{g}_j$, and vectors and tensors are denoted using bold letters.