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An explicit time-domain approach for sensitivity analysis of non-stationary random vibration problems

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ABSTRACT

This paper presents an explicit time-domain method for sensitivity analysis of structural responses under non-stationary random excitations. Based on time-domain explicit expressions of dynamic responses, a new and more concise time-domain explicit expression of response sensitivity is derived using the direct differentiation method (DDM). Then a more efficient algorithm for direct construction of the explicit expression of response sensitivity is developed based on the physical meanings of the coefficient matrices in the formulation. The adjoint variable method (AVM) is further used to establish the explicit expression of the sensitivity of an arbitrary response. Finally, based on the time-domain explicit expressions for both dynamic response and its sensitivity, an efficient time-domain approach is proposed to calculate the sensitivity of variance responses of a structure subjected to non-stationary random excitations. Numerical examples of different structural systems under non-stationary random excitations are presented to demonstrate the accuracy and efficiency of the proposed method.

1. Introduction

Most dynamic excitations arising from natural environment, such as earthquake, wind and wave, are random loads commonly modeled as random processes. The random vibration theory can be used to deal with the problem that a structure is subjected to such a random excitation [1,2]. In the field of structural optimization and optimal control, the development of effective sensitivity analysis methods of random responses is of great significance and has attracted growing research interest.

According to their statistical characteristics, random excitations can be classified into stationary and non-stationary processes. Research on sensitivity analysis methods is now comparatively mature, and a large number of related studies have been reported, including those on correlated and un-correlated generalized random excitations [3], Gaussian and non-Gaussian response sensitivity [4] and uncertain structures [5]. In contrast, there is limited work published on the sensitivity analysis of non-stationary responses due to the complexity of the random vibration analysis. Chaudhuri and Chakraborty [6] dealt with the evaluation of response sensitivity in the frequency domain of structures subjected to non-stationary seismic excitations. The sensitivity of reliability of structure with respect to depths of columns and beams is calculated. Cacciola

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et al. [7] proposed a semi-analytical approach for determining the sensitivity of the random responses of both classically and non-classically damped structural systems subjected to stationary and non-stationary random Gaussian excitations. The equations governing the sensitivity of the statistical moments of response are derived in the time domain. Marano et al. [8] performed the sensitivity analysis of response spectra with respect to uncertain soil parameters in the presence of a non-stationary seismic excitation. Based on the pseudo-excitation method [9], a new approach for first and second order sensitivity analysis of non-stationary random responses and evolutionary power spectral density functions has been presented [10,11].

Recently, Su and Xu [12] proposed an explicit time-domain method for random vibration analysis. The statistical moments can be evaluated directly using time-domain explicit expressions of dynamic responses, and the computational efficiency and accuracy have been shown through numerical examples of large-scale structures [13]. Based on this explicit time-domain method, the expressions for calculating the sensitivity of a non-stationary random response has also been derived in the state space, and the effectiveness of the proposed method is demonstrated in [14].

The main purpose of this paper is to present an explicit time-domain method for sensitivity analysis of variance responses of structures subjected to non-stationary random excitations. Based on the earlier study in [14], a new and more concise time-domain explicit expression of response sensitivity is derived using DDM. Then a more efficient algorithm for direct calculation of the coefficient matrices in the explicit expression of response sensitivity is derived based on the physical meanings of the matrices. For sensitivity problems with a large number of structural parameters, the AVM is further employed to construct the explicit expression for sensitivity analysis of an arbitrary response. Finally, using the two sets of explicit expressions derived for the dynamic response and its sensitivity as well, an efficient time-domain method is proposed for sensitivity analysis of variance responses of a structure under non-stationary random excitations.

In the proposed method, the time-varying sensitivities of the statistical moments of non-stationary structural responses can be obtained directly in the time domain, and thus the present approach is more efficient as compared with the mixed time-frequency-domain approach based on the power spectrum method [6,10,11], in which a large number of time-history integrals are required at different frequency intervals within the range of frequency concerned when non-stationary random excitations are involved. Furthermore, in the process of variance sensitivity analysis by the proposed method, one can focus on the specific responses concerned without the need to consider the whole system, which means analysis dimension can be easily reduced in the calculation of variance sensitivity owing to the use of the explicit expressions of dynamic response and its sensitivity. Numerical examples are presented to demonstrate the feasibility of the present approach.

2. Time-domain explicit expressions of dynamic responses

As formulations of the sensitivity analysis methods to be discussed in this paper are based on the explicit time-domain method for random vibration analysis [12,13], the time-domain explicit expressions of dynamic responses are summarized firstly to facilitate discussions in the following sections.

2.1. The general case

For a generic linear dynamic structural system, the equation of motion can be expressed in the following form

$$\mathbf{M}\ddot{\mathbf{u}}(t)+\mathbf{C}\dot{\mathbf{u}}(t)+\mathbf{K}\mathbf{u}(t)=\mathbf{P}(t) \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$and $\mathbf{K}$ are mass, damping and stiffness matrices of the system; $\ddot{\mathbf{u}}(t)$, $\dot{\mathbf{u}}(t)$ and $\mathbf{u}(t)$ are acceleration, velocity and displacement vectors; $\mathbf{P}(t)$ is a time-dependent loading vector.

It is assumed that a numerical time integration scheme is used to solve the dynamic equation with given initial conditions $\mathbf{u}_0=\mathbf{u}(t_0)$ and $\dot{\mathbf{u}}_0=\dot{\mathbf{u}}(t_0)$. Responses at discrete time instants, including $\mathbf{u}_i=\mathbf{u}(t_i)$, $\dot{\mathbf{u}}_i=\dot{\mathbf{u}}(t_i)$ and $\ddot{\mathbf{u}}_i=\ddot{\mathbf{u}}(t_i)$ $(i=1,2,\cdots,N)$, can be determined for a given set of loading vectors $\mathbf{P}_i=\mathbf{P}(t_i)$ $(i=0,1,2,\cdots,N)$, where N is the number of time steps and $t_i=i\Delta t$ with Δt being a constant time step. A numerical solution using one of the general transient dynamic analysis methods [15] can be expressed as

$$\begin{aligned}\mathbf{u}_i&=\mathbf{T}_{11}\mathbf{u}_{i-1}+\mathbf{T}_{12}\dot{\mathbf{u}}_{i-1}+\mathbf{Q}_{11}\mathbf{P}_{i-1}+\mathbf{Q}_{12}\mathbf{P}_i\\ \dot{\mathbf{u}}_i&=\mathbf{T}_{21}\mathbf{u}_{i-1}+\mathbf{T}_{22}\dot{\mathbf{u}}_{i-1}+\mathbf{Q}_{21}\mathbf{P}_{i-1}+\mathbf{Q}_{22}\mathbf{P}_i\end{aligned} \tag{2}$$

where $\mathbf{T}_{pq}$ and $\mathbf{Q}_{pq}(p,q=1,2)$ are matrices dependent on matrices of the system, time step Δt, the integration scheme and the related parameters used. When using the Newmark-β integration approach, $\mathbf{T}_{pq}$ and $\mathbf{Q}_{pq}(p,q=1,2)$ can be derived as [16]

$$\begin{aligned}\mathbf{T}_{11}&=\overline{\mathbf{K}}^{-1}\left[(a_0\mathbf{M}+a_3\mathbf{C})-(a_2\mathbf{M}+a_5\mathbf{C})\mathbf{M}^{-1}\mathbf{K}\right]\\ \mathbf{T}_{12}&=\overline{\mathbf{K}}^{-1}\left[(a_1\mathbf{M}+a_4\mathbf{C})-(a_2\mathbf{M}+a_5\mathbf{C})\mathbf{M}^{-1}\mathbf{C}\right]\\ \mathbf{T}_{21}&=a_3(\mathbf{T}_{11}-\mathbf{I})+a_5\mathbf{M}^{-1}\mathbf{K}\\ \mathbf{T}_{22}&=a_3\mathbf{T}_{12}-a_4\mathbf{I}+a_5\mathbf{M}^{-1}\mathbf{C}\end{aligned} \tag{3}$$