Journal of Sound and Vibration ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

A hybrid approach to analyse a beam-soil structure under a moving random load

L.T. Si [a], Y. Zhao [a,*], Y.H. Zhang [a], D. Kennedy [b]

[a] State Key Laboratory of Structural Analysis for Industrial Equipment, Faculty of Vehicle Engineering and Mechanics, Dalian University of Technology, Dalian 116023, PR China
[b] Cardiff School of Engineering, Cardiff University, Cardiff CF24 3AA, Wales, UK

ARTICLE INFO

Article history:
Received 13 October 2015
Received in revised form
3 July 2016
Accepted 7 July 2016
Handling Editor: L. G. Tham

Keywords:
Moving random load
Pseudo-excitation method
Fourier transform
Wavelet method

ABSTRACT

To study the stochastic response of a beam-soil structure under a moving random load, a hybrid approach based on the pseudo-excitation method and the wavelet method is proposed. Using the pseudo-excitation method, the non-stationary random vibration analysis is transformed into a conventional moving harmonic load problem. Analytical solutions of the power spectral density and standard deviation of vertical displacement are derived in an integral form. However, the integrand is singular and highly oscillatory, and the computational time is an important consideration because a large number of frequency points must be computed. To calculate the response accurately and efficiently, a wavelet approach is introduced. Numerical results show that the frequency band which brings the most significant response is dependent on the load velocity. The hybrid method provides a useful tool to estimate the ground vibration caused by traffic loads.

1. Introduction

Trains travelling at speed present moving loads that are known to excite large amplitude, wide frequency spectrum vibration in the track which can propagate over long distances. Such vibrations can enter buildings through their foundations and affect the operation of sensitive equipment and human comfort. The moving loads are random in nature, due in part to rail irregularities.

Several works have previously been conducted to solve the deterministic problems of a half-space subjected to moving loads. The models have become complicated, and various solution methods have emerged. Lamb [1] first proposed the problem of an elastic medium subjected to an impulsive force. Eason [2] extended the problem by considering a moving force acting on a uniform half-space. Gakenheimer and Miklowitz [3] derived the transient displacements inside the half-space induced by a moving load, while Fryba [4] analysed the steady-state response of an unbounded elastic half-space under a moving load. Each work considered the effect of the moving load velocity on the response of the half-space and studied the subcritical, critical and supercritical cases. In later studies, a uniform or layered half-space was subjected to different types of moving loads including elastically distributed loads [5–7], normal or shear stresses [8], harmonic rectangular [9,10] or strip loads [11,12] and vehicle loads [13]. Using a layered half-space model coupled with a track structure

* Corresponding author. Fax: +86 411 8470 6337.
E-mail address: yzhao@dlut.edu.cn (Y. Zhao).

http://dx.doi.org/10.1016/j.jsv.2016.07.012

subjected to a fixed or moving harmonic load, Sheng et al. [14–16] theoretically investigated the propagation of the vibration.

However, in actual rail transportation systems, the moving loads caused by vehicles are somewhat stochastic due to the track irregularity and other uncertainties. As a result, the vibration at any specific ground location is a non-stationary random process. Hunt [17,18] computed the power spectrum of ground vibration under traffic loads by assuming the ground vibration was a random and statistically stationary process. Sun and Greenberg [19] presented a generalised method to solve the problem of a linear system subjected to moving sources. Metrikine and Vrouwenvelder [20] studied the steady-state surface ground vibration under a point load moving along a beam embedded in a layer, in which a stationary random load was investigated, with the randomness represented by a uniformly distributed phase angle. Sheng et al. [21] investigated ground vibration considering vertical track irregularities, but the power spectral density (PSD) was not time-varying because the fast Fourier transform with respect to time was applied. By using the pseudo-excitation method (PEM), Lu et al. [22] adopted a model similar to Sheng's to study the random response due to random moving loads in the subcritical case. Considering the PSD of the track irregularity, Lombaert and Degrande [23] studied the random response of a track-soil system under dynamic excitation in the subcritical case.

Significant work remains to solve the problems of a beam-soil structure under random moving loads, especially in the critical and supercritical cases. Obtaining the second-order statistics of the response using traditional methods is time consuming, and the highly oscillatory nature of the integrand can cause large errors in numerical results. The objective of this paper is to present a convenient method to study the non-stationary stochastic vibration of a beam-soil structure subjected to a moving random load. A hybrid method based on the PEM and wavelet method is proposed. In Section 2, the basic model and governing equations are provided. In Section 3, the PEM used in a linear system is introduced. An analytical solution is given in Section 4 and the wavelet approach to compute the integrands is shown in Section 5. Sections 6 and 7 give numerical results, discussion and conclusions.

2. Model and governing equations

Fig. 1 depicts a beam-soil structure consisting of an infinitely long beam located on the surface of a homogenous isotropic visco-elastic half-space and subjected to a random load $p(t)$ moving with velocity V.

The vertical motion of the beam is described by the Euler-Bernoulli equation

$$EI\frac{\partial^4 W(x,t)}{\partial x^4}+\rho_B\frac{\partial^2 W(x,t)}{\partial t^2}=p(t)\delta(x-Vt)-a\sigma_{zz}(x,0^+,t) \tag{1}$$

where $W(x,t)$ is the vertical displacement of the beam, $\sigma_{zz}(x,z,t)$ is the vertical stress, EI and ρ_B are the bending stiffness and mass per unit length of the beam, respectively, a is its thickness in the y direction and $\delta(\bullet)$ is the Dirac delta function.

The motion of the soil is modelled by considering a small viscosity in the soil using the elastodynamic Navier's equation

$$\left(\hat{\lambda}+\hat{\mu}\right)\nabla_{xz}(\nabla_{xz}\bullet\mathbf{u})+\hat{\mu}\nabla^2_{xz}\mathbf{u}=\rho\frac{\partial^2\mathbf{u}}{\partial t^2} \tag{2}$$

where $\hat{\lambda}=\lambda+\lambda^*\partial/\partial t$ and $\hat{\mu}=\mu+\mu^*\partial/\partial t$ describe the visco-elastic behaviour of the soil, λ and μ are Lamé constants derived from its elastic modulus E and Poisson's ratio ν, λ^* and μ^* are visco-elastic constants, ρ is the mass density of the soil and $\mathbf{u}(x,z,t)=\{u(x,z,t),w(x,z,t)\}^T$ is the time-dependent displacement vector.

It is assumed that the beam does not move horizontally and that the displacement of the beam and the soil are the same at the interfaces. Therefore, the boundary and continuity conditions can be written as

$$u(x,0,t)=0 \qquad w(x,0,t)=W(x,t) \tag{3a}$$

$$\lim_{z\to\infty}u(x,z,t)=0 \qquad \lim_{z\to\infty}w(x,z,t)=0 \tag{3b}$$

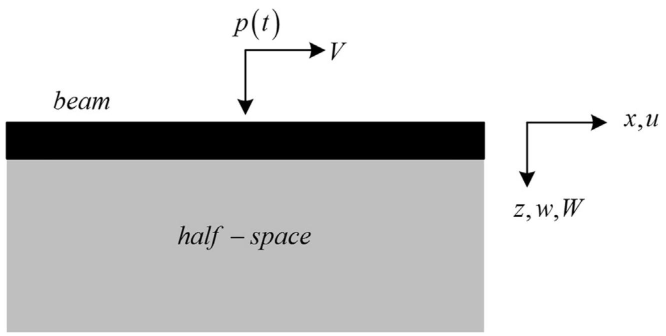

Fig. 1. Beam-soil structure under a moving random load.

Please cite this article as: L.T. Si, et al., A hybrid approach to analyse a beam-soil structure under a moving random load, *Journal of Sound and Vibration* (2016), http://dx.doi.org/10.1016/j.jsv.2016.07.012