Journal of Sound and Vibration 382 (2016) 213–226

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

An efficient quadrature for 2.5D boundary element calculations

Christian H. Kasess *, Wolfgang Kreuzer, Holger Waubke

Acoustics Research Institute, Austrian Academy of Sciences, Wohllebengasse 12-14, 1040 Vienna, Austria

ARTICLE INFO

Article history:
Received 18 December 2015
Received in revised form
20 May 2016
Accepted 29 June 2016
Handling Editor: J. Astley
Available online 28 July 2016

Keywords:
Quadrature
BEM
2.5D
Noise Barriers

ABSTRACT

In recent years, the boundary element method has become a widely used tool for calculating the mitigation effects of noise barriers. However, since for large structures calculations in 3D become very inefficient, most of the standard implementations are only in 2D. This means that the noise source is implicitly assumed to be given by a coherent line source, which is not realistic in most cases. By using a Fourier transform with respect to a spatial coordinate along the length of the structure it is possible to reduce the 3D problem to several 2D problems with distinct wavenumbers which allows the simulation of more realistic noise sources and which is typically referred to as 2.5D BEM. To that end, it is necessary to numerically calculate a Fourier-like integral over all the 2D solutions. In this work, an efficient way to calculate this integral is given building on existing approaches using Clenshaw–Curtis–Filon quadrature and demodulation combined with an adaptive order-selection scheme. As BEM calculations are costly, the main focus of the method introduced lies on avoiding too many of these calculations. The efficiency of the method is illustrated using two different examples: a reflecting cylinder and an L-shaped noise barrier.

1. Introduction

The boundary element method (BEM) is an attractive tool for modeling outdoor sound propagation because only the surface of the scatterer needs to be discretized and (semi)infinite domains can be treated very efficiently. Especially for the calculation of mitigation effects of noise barriers, the application of the BEM has received growing attention over the last two decades (see e.g. [1–15]). The length of noise barriers can be anywhere between a few hundred meters to many kilometers. Even though methods exist to considerably speed up the calculation for large BEM-problems such as the fast multipole method or $\mathcal{H}$-matrices (cf. [16,17]) a full simulation of such a large structure in 3D is not feasible, in particular at high frequencies, as the number of BEM elements needed for accurate calculations scales quadratically with the frequency. Simply using a shorter barrier segment to model the full barrier leads to a perturbation of the real solution by an unwanted diffraction of the sound around the barrier ends. This effect becomes increasingly problematic when the receiver or the source is placed far away from the barrier or in the vicinity of the end of the barrier which may happen, for instance, when modeling oblique incidence.

* Corresponding author.
E-mail address: christian.kasess@oeaw.ac.at (C.H. Kasess).

http://dx.doi.org/10.1016/j.jsv.2016.06.041

Although approaches exist to reduce the computational effort in 3D by harnessing certain symmetries in the structure such as periodicity (see e.g. [14]), it is in most practical applications typically assumed that the noise barrier has a constant cross-section and is straight along its length as well as infinitely long. Under these assumptions the full 3D calculations can be reduced to 2D to enhance numerical efficiency (for details see e.g. [18]). In that case, however, it is also implicitly assumed that the sound source itself is infinitely long and fully coherent along its length, i.e. the sound source is represented by infinitely many point sources lying along a straight and infinitely long line that oscillate in phase. Besides being clearly not realistic it has been shown that the source type can have a considerable effect on the insertion loss of a barrier [19] and thus this simplification may lead to wrong results. To circumvent this problem, Duhamel [20] published a method for a 2.5D calculation of the acoustic field for infinitely long objects of constant cross-section that allows using point sources (for applications see e.g. [5,13]). This method reduces a 3D problem to *multiple* 2D problems and requires the calculation of an inverse Fourier transform based on the 2D BEM solutions $p_2(x,y,\kappa)$ at different wavenumbers $\kappa=\sqrt{K^2-\alpha^2}$:

$$p_3(x,y,z,K)=\frac{1}{2\pi}\int_{-\infty}^{\infty}p_2(x,y,\kappa)\mathrm{e}^{\mathrm{i}\alpha z}\mathrm{d}\alpha, \tag{1}$$

where $p_3(x,y,z,K)$ is the sound pressure of the three-dimensional scattering problem at the receiver node (x,y,z) at the desired wavenumber K in 3D. α denotes the wavenumber of the oscillation along the infinite dimension of the scatterer. The 2D solutions $p_2(x,y,\kappa)$ have an oscillatory behavior depending on the distance r between the sound source and the receiver in the x–y-plane. This oscillation is, contrary to the Fourier oscillation term $\mathrm{e}^{\mathrm{i}\alpha z}$, not linear in the variable α. In [20], p_2 was interpolated with piecewise polynomials of degree 2. To have sufficient data points for an accurate interpolation an estimation for the minimum frequency resolution of 5 BE calculations per period in κr was given. As an example, for a frequency of 2000 Hz and a source–receiver distance of 50 m roughly 1500 BE problems need to be solved. In the original work of Duhamel it was suggested that the interpolation may require less points if the demodulated function is interpolated using the relation between the 2D-wavenumber κ, the integration variable α, and the source–receiver distance as a basis for demodulation. The potential reduction in computational effort due to the demodulation has, to our knowledge, not been fully investigated in the context of the 2.5D BEM yet.

In this work, we will use this demodulation approach and include it in an efficient quadrature scheme that deals analytically with the oscillatory terms without the need for explicit interpolation. For this, a number of existing algorithms are combined and adapted to deal with the specific challenges that appear in connection with the 2.5D BEM.

In the following, the properties of the integral used in the 2.5D Helmholtz BEM will be outlined and the different components of the quadrature scheme will be described. This encompasses a discussion about a suitable quadrature algorithm for highly oscillatory integrands, methods to efficiently deal with the nonlinear oscillations of p_2, an adaptive order selection scheme, as well as a global error estimate. The properties and the efficiency of the new quadrature scheme are investigated using two examples: a fully reflecting cylinder for which a semi-analytic expression exists and a noise barrier.

2. Methods

2.1. The 2.5D boundary element method

In 1996, Duhamel [20] introduced a 2.5D method that allows to calculate the scattering effect of an infinitely long structure caused by a point source. Essentially, the method utilizes a representation of the 3D-Green's function by its Fourier transform:

$$\frac{\mathrm{e}^{\mathrm{i}K\sqrt{\|\mathbf{x}-\mathbf{y}\|^2+z^2}}}{4\pi\sqrt{\|\mathbf{x}-\mathbf{y}\|^2+z^2}}=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\mathrm{i}}{4}H_0\left(\sqrt{K^2-\alpha^2}\,\|\mathbf{x}-\mathbf{y}\|\right)\mathrm{e}^{\mathrm{i}\alpha z}\mathrm{d}\alpha, \tag{2}$$

where H_0 denotes the Hankel function of the first kind:

$$H_0(\kappa\|\mathbf{x}-\mathbf{y}\|)=J_0(\kappa\|\mathbf{x}-\mathbf{y}\|)+\mathrm{i}Y_0(\kappa\|\mathbf{x}-\mathbf{y}\|). \tag{3}$$

$\mathbf{x}$ and $\mathbf{y}$ denote two points in the x–y-plane and $\|\mathbf{x}-\mathbf{y}\|$ is the distance between those points. Conceptually, Eq. (2) can be seen as a superposition of infinitely many line sources that exhibit oscillations along their length with different wavenumbers α.

As a consequence, the 3D-problem can be reduced to a number of 2D-problems for different wavenumbers $\kappa=\sqrt{K^2-\alpha^2}$, which are solved numerically using the BEM. The solution of the 3D-problem is then given by the inverse Fourier transform over the 2D-solutions for different values of α (Eq. (1)). As BEM calculations are costly, it is necessary to design a quadrature method for the back transform that keeps the number of evaluations of $p_2(x,y,\sqrt{K^2-\alpha^2})$ small.