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A dynamic model of a cantilever beam with a closed, embedded horizontal crack including local flexibilities at crack tips

J. Liu [a,b,c,*], W.D. Zhu [c,**], P.G. Charalambides [c], Y.M. Shao [a], Y.F. Xu [c], X.M. Fang [c]

[a] *State Key Laboratory of Mechanical Transmission, Chongqing University, Chongqing 400030, PR China*
[b] *College of Mechanical Engineering, Chongqing University, Chongqing 400030, PR China*
[c] *Department of Mechanical Engineering, University of Maryland Baltimore County, Baltimore, MD 21250, USA*

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ABSTRACT

As one of major failure modes of mechanical structures subjected to periodic loads, embedded cracks due to fatigue can cause catastrophic failure of machineries. Understanding the dynamic characteristics of a structure with an embedded crack is helpful for early crack detection and diagnosis. In this work, a new three-segment beam model with local flexibilities at crack tips is developed to investigate the vibration of a cantilever beam with a closed, fully embedded horizontal crack, which is assumed to be not located at its clamped or free end or distributed near its top or bottom side. The three-segment beam model is assumed to be a linear elastic system, and it does not account for the nonlinear crack closure effect; the top and bottom segments always stay in contact at their interface during the beam vibration. It can model the effects of local deformations in the vicinity of the crack tips, which cannot be captured by previous methods in the literature. The middle segment of the beam containing the crack is modeled by a mechanically consistent, reduced bending moment. Each beam segment is assumed to be an Euler–Bernoulli beam, and the compliances at the crack tips are analytically determined using a *J*-integral approach and verified using commercial finite element software. Using compatibility conditions at the crack tips and the transfer matrix method, the nature frequencies and mode shapes of the cracked cantilever beam are obtained. The three-segment beam model is used to investigate the effects of local flexibilities at crack tips on the first three natural frequencies and mode shapes of the cracked cantilever beam. A stationary wavelet transform (SWT) method is used to process the mode shapes of the cracked cantilever beam; jumps in single-level SWT decomposition detail coefficients can be used to identify the length and location of an embedded horizontal crack.

1. Introduction

Embedded cracks are one of major failure modes of mechanical structures subjected to periodic loads. The dynamic characteristics and the safety of machineries are greatly affected by cracks due to fatigue. To prevent catastrophic failure of

** Corresponding author. Tel.: +1 410 455 3394.
* Corresponding author at: State Key Laboratory of Mechanical Transmission, Chongqing University, Chongqing 400030, PR China. Tel.: +86 23 65112520.
E-mail addresses: jliu@cqu.edu.cn (J. Liu), wzhu@umbc.edu (W.D. Zhu).

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machineries, mechanical structure monitoring for early crack detection and diagnosis is an important task for industrial maintenance.

When a crack occurs in a structure, its static and dynamic characteristics such as the stiffness, natural frequencies, mode shapes, damping, and vibration amplitudes will be changed [1,2]. An investigation of changes in the static and dynamic characteristics makes it possible to detect a crack in a structure [1–4]. While mode shapes and damping are more sensitive to the existence of a crack in a structure than natural frequencies in practice, magnitudes of changes in the natural frequencies are also functions of the severity and location of the cack in the structure [2,5–10]. Many research works determine damage severity of beam structures using analytical, numerical, and experimental methods. For an open crack, damage detection depends on changes in the static and dynamic characteristics. For a breathing crack, it depends on nonlinear dynamic characteristics, such as periodical structural stiffness variation, modulation frequencies, and higher harmonics, which was discussed in Ref. [11].

As one of accurate and comprehensive methods, a modeling and simulation method can predict the dynamic characteristics of a cracked structure and provide some guidance to early detection of crack failure. Many previous works focused on an edge crack [9,12–24] and multiple edge cracks [3,10,25–30] in cantilever or simply supported beams. Some researchers have studied delaminations in beam structures [16,31–46]. While some studies [33,34,41,43,46] are focused on detection of delamination in a laminated material, some [4,16,31,32,35–38,40,42,44,45] are focused on modeling delaminations in beam structures based on analytical methods, the finite element (FE) method, and experimental methods. The compatibility conditions at the junctions are formulated as changes in the axial forces and bending moments there [16,31,32,35–37,39,40,42,44], which cannot describe local flexibilities at crack tips due to the presence of a crack. Wang and Qiao [38] used a shear compliance coefficient at a crack tip to describe the crack tip deformation for a simply-supported end-notched beam specimen. Qiao and Chen [45] used the model in Ref. [38] to study deformations at delamination tips in a clamped and a simply supported bi-layer composite beam with an interface delamination. Cantilever beam structures with embedded cracks have not been discussed in the literature.

In this work, a new three-segment beam model with local flexibilities at crack tips is developed to investigate the vibration of a cantilever beam with a closed, fully embedded horizontal crack, which is assumed to be not located near its clamped or free end or distributed near its top or bottom side. For the closed, fully embedded horizontal crack, the top and bottom segments always stay in contact at their interface and have the same transverse displacements; they can slide over each other in the axial direction except at their ends. Such a crack can occur in a layered structure prone to delamination. While the beam is assumed to be homogeneous here, the methodology developed in this work can be extended to laminates with homogeneous layers, whose material properties are not functions of spatial variables. Hence, the three-segment beam model is assumed to be a linear elastic system and does not account for the nonlinear crack closure effect. The proposed model can describe the effects of local deformations in the vicinity of the crack tips, which cannot be captured by previous analytical methods in the literature. The middle segment of the beam containing the crack has a mechanically consistent, reduced bending moment. This work builds on parallel studies in Ref. [47] where the macro-mechanics of a cantilever beam with an embedded horizontal crack under a static load is addressed. Each beam segment is assumed to be an Euler–Bernoulli beam in this work, which implies that it is slender. However, the methodology developed here can be extended to the case where each beam segment is modeled as a Timoshenko beam. Compliances at the crack tips are analytically determined using a J-integral approach [48]. The J-integral approach may not be used to analyze a breathing, embedded crack with opening and closing states, or a crack located near the clamped or free end of the beam, or distributed near its top or bottom side. Using compatibility conditions at the crack tips and the transfer matrix method [18,27,49], the natural frequencies and mode shapes of the cracked cantilever beam are obtained. Since the FE method has been widely used in deformation and vibration studies of beams with cracks [1,22,42,50–56], the J-integral and stress state results from the analytical method are verified using commercial FE software [57]. A more detailed comparison of the J-integral results using the analytical and FE methods is presented in Ref. [47].

The new three-segment beam model is used to investigate the effects of local flexibilities at crack tips on the first three natural frequencies and mode shapes of the cracked cantilever beam. The results show that the model put forward here is an improvement over the related one, where crack-induced rotational flexibilities of cross-sections of the beam at the crack tips are not considered. As will be demonstrated later in this study, inclusion of local flexibilities at the crack tips can model

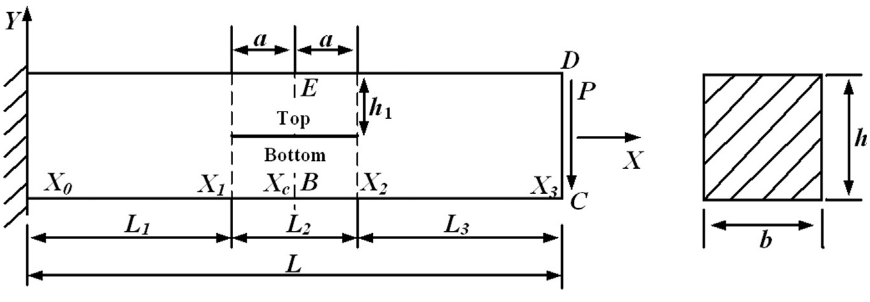

Fig. 1. Schematic of a cantilever beam with a closed, fully embedded horizontal crack.