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Equivalent material modelling of sandwich beams, evanescent solutions and damping investigations

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ABSTRACT

A novel method for representing the transverse vibrations of sandwich beams as equivalent Timoshenko beams is developed. Special attention is given to damping modelling together with the evanescent parts of the solutions to assert applicability of the approach to any boundary conditions. Shear stiffness is evaluated based on current knowledge. The latter is then used to update the reference theory for vibrations in sandwich beams. Analytical case studies are presented to show the performance and limitations of the method and compared with experimental data.

1. Introduction

With increasing interest in engineering of fuel efficient vehicles, lightweight structures presenting high stiffness to mass ratios are an essential component of novel design in the automobile, train and aerospace industries. Sandwich structures are a fitting answer to lightweight constructions requiring both high bending stiffness and low mass. The dynamic behaviour of such structures is more complex than single material components due to their multilayered construction and therefore need to be first well understood, and then modelled. For design and analysis engineers, fast and comprehensive methods need to be developed for accurate and time efficient modelling of these novel structures. Specifically for Finite Element Analysis (FEA), representing the multilayered system as an equivalent single layered material using shell elements with acceptable accuracy over the frequency range of interest would greatly accelerate computing times and therefore allow for more design iterations. While the idea to model sandwich beams and plates as equivalent single materials is not new, previous research has focussed on respecting the propagating part of the solution and typically matching eigenfrequencies of simply supported components. In practical applications, the evanescent parts of the solution are however just as important. Since they are local deformation fields at the boundaries, they influence the loads transmitted at the interfaces between parts of the structure. In addition, as will be shown in the following sections, the evanescent fields turn out to have a substantial influence on the total energy dissipated due to material losses. They hence play an important role in the damping which is assigned to the equivalent material. In this paper the focus is on the transverse vibrations of sandwich beams.

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This paper aims at first gathering the current knowledge on the dynamics of sandwich and Timoshenko beams which is presented in Section 2.1. Existing methods to better qualify the shear stiffness of laminates are presented in Section 2.2. The state of the art of damping modelling is presented in Section 2.3. Then, based on the gathered knowledge, together with updated considerations concerning evanescent fields, shear stiffness and damping, a new methodology is presented for choosing material properties to construct an equivalent Timoshenko beam in Section 3. Subsequently, using analytically treated examples, the virtues and limitations of the methodology, especially for the equivalent damping coefficient developed, are highlighted in Section 4. The accuracy of the equivalent material model is mainly evaluated based on the eigenfrequencies' location, as well as the width of the peaks in the frequency responses computed for the modelled beams. Finally, our method is compared to experimental data in Section 5, revealing the importance of the bonding layer between the face sheets and the core both for stiffness and damping characteristics.

2. State of the art in sandwich beams and plates dynamics

2.1. Equations of motion

A substantial amount of research has been conducted over the past few decades to model the transverse vibrations of sandwich beams and plates. Several existing theories have therefore been further developed, covering a wide range of more or less complex models. The simplest, but also least accurate theory employed, is classic theory (Kirchhoff for plates, Euler when considering beams) which only considers pure bending. A more popular choice is based on first-order shear theory (Mindlin for plates and Timoshenko for beams) where transverse shear and rotary inertia terms are included. Higher level of detail can be achieved by using higher order theories, at the cost of heavier calculations. A couple of examples of previous work that can be found in the literature are listed here. Yang et al. [1] developed general equations of motion for orthotropic laminates, where plane stress is assumed in each layer and the global displacements of the plate are represented by first-order shear theory. A simplification to classic plate theory is also presented. Integration of the material properties of each layer over the cross section give the stress–strain relations of the laminate. Solutions for the eigenfrequencies of simply supported symmetric sandwich plates are presented by Yu and Cleghorn [2], who construct the stiffness and mass matrices of the system following at first classic plate theory, then Mindlin plate theory and finally Reddy's third-order plate theory. The stress–strain relations are calculated following the same approach as in [1]. Wang [3] applied classic plate theory solutions of simply supported sandwich plates into an equivalent Mindlin plate. The bending and shear stiffnesses of the plate were obtained as the sum of the stiffnesses of each layer. Maheri and Adams [4] and Lok and Cheng [5] model sandwich plates following Mindlin plate theory where the stiffness terms are obtained from static stiffness calculations on a small representative section of the sandwich structure. Maheri and Adams [4] conducted their study on free–free beams. Lok and Cheng [5] calculated the eigenfrequencies of the Mindlin plate assuming simply supported boundary conditions. The works of Mead [6], Zhou and Li [7], Banerjee and Sobey [8] and Banerjee et al. [9] are examples where the motion of the face sheets and core are first considered separately following their respectively relevant plate equations and then coupled together into a single set of plate equations. Gaudenzi and Carbonaro [10], Kant and Swaminathan [11] and Matsunaga [12] are examples of higher order theory representations of sandwich structures. Several more detailed reviews can be found on this topic, such as the reviews of Mallikarjuna and Kant [13] or Denli and Sun [14].

In this paper, we treat specifically beam cases for the sake of simplicity and to have the freedom of having analytical solutions for any boundary condition, which is not the case for plates. We consider here the governing equations given by Nilsson and Nilsson [15] as the reference theory as it has been developed specifically for sandwich beams. In this theory, the total bending stiffness of the layup D_{tot}, the transverse shear stiffness of the core D_s and the individual bending stiffness of the face sheets about their own mid-plane D_f are considered and given by:

$$D_{tot} = \frac{E_c H_c^3}{12} + E_f\left(\frac{H_c^2 h_f}{2} + H_c h_f^2 + \frac{2h_f^3}{3}\right), \quad D_s = G_c H_c, \quad D_f = \frac{E_f h_f^3}{12} \tag{1}$$

with E_c and E_f the elasticity moduli of the core and face sheets, H_c and h_f the thicknesses of the core and the face sheets and G_c the transverse shear modulus of the core.

The rotary and transverse inertias, I_ρ and μ, also considered here are given by:

$$I_\rho = \frac{\rho_c H_c^3}{12} + \rho_f\left(\frac{H_c^2 h_f}{2} + H_c h_f^2 + \frac{2h_f^3}{3}\right), \quad \mu = 2h_f\rho_f + H_c\rho_c \tag{2}$$

with ρ_c and ρ_f the densities of the core and the face sheets.

Using the Hamiltonian approach, the governing equations are obtained as follows:

$$-D_s\left(\frac{\partial^2 W}{\partial x^2} - \frac{\partial \psi}{\partial x}\right) + 2D_f\left(\frac{\partial^4 W}{\partial x^4} - \frac{\partial^3 \psi}{\partial x^3}\right) + \mu\frac{\partial^2 W}{\partial t^2} = 0 \tag{3}$$

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