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Cascade trailing-edge noise modeling using a mode-matching technique and the edge-dipole theory

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ABSTRACT

An original analytical approach is proposed to model the broadband trailing-edge noise produced by high-solidity outlet guide vanes in an axial turbomachine. The model is formulated in the frequency domain and first in two dimensions for a preliminary assessment of the method. In a first step the trailing-edge noise sources of a single vane are shown to be equivalent to the onset of a so-called edge dipole, the direct field of which is expanded in a series of plane-wave modes. A criterion for the distance of the dipole to the trailing-edge and a scaling of its amplitude is defined to yield a robust model. In a second step the diffraction of each plane-wave mode is derived considering the cascade as an array of bifurcated waveguides and using a mode-matching technique. The cascade response is finally synthesized by summing the diffracted fields of all cut-on modes to yield upstream and downstream sound power spectral densities. The obtained spectral shapes are physically consistent and the present results show that upstream radiation is typically 3 dB higher than downstream radiation, which has been experimentally observed previously. Even though the trailing-edge noise sources are not vane-to-vane correlated their radiation is strongly determined by a cascade effect that consequently must be accounted for. The interest of the approach is that it can be extended to a three-dimensional annular configuration without resorting to a strip theory approach. As such it is a promising and versatile alternative to previously published methods.

1. Introduction

The design of many axial-flow fan stages involves a rotor and a downstream row of stationary outlet guide vanes called stators as shown in Fig. 1. The aerodynamic noise of the rotor–stator stage is caused by various aerodynamic interactions, all responsible for unsteadiness of the velocity relative to the blades/vanes. According to Ffowcs Williams and Hawkings' acoustic analogy and for subsonic Mach numbers, sound mainly originates from the fluctuating lift forces that result from this unsteadiness, all acting as equivalent dipoles [1].

The main manifestation of this generic mechanism is the wake-interaction noise produced as the wakes of the rotor blades impinge on the stator vanes. The mean velocity deficit and the turbulence in the wakes generate tonal noise and broadband noise, respectively, the sources of which are distributed on the vanes. Similarly, stationary inflow distortions and ingested turbulence at inlet are the origin of tonal and broadband noise generated by the rotor blades. Independently the

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Nomenclature

a	constant
$\overline{a}_j, a_j$	pressure and potential coefficients for plane waves
A^j_m, B^j_s	transmitted and reflected mode amplitudes, single interface
c	chord length
c_0	sound speed
D^0_m, U^0_m	downstream and upstream mode amplitudes in the channels
E, F	Fresnel integral and related function
h	inter-vane channel height
$k=\omega/c_0$	acoustic wavenumber
$K=k/\beta$	scaled wavenumber
$K^{(j)}$	axial wavenumber in the channels
K_m	incident axial wavenumber
$\overline{K}^j_s$	effective axial wavenumbers of reflected/transmitted waves
ℓ_y	wall-pressure spanwise coherence length
L	vane spanwise extent
M_0	axial Mach number
p, p_0	acoustic pressure
r_c	radius of the unwrapped cut of the stator
$(r_0,\theta_0),(r,\theta)$	source and observer cylindrical coordinates around an edge
$(\overline{r}_0,\overline{\theta}_0),(\overline{r},\overline{\theta})$	same coordinates corrected for convection
$\overline{R}$	corrected source-to-observer distance
R_s, T_s	reflected and transmitted mode amplitudes, double interface
R_1, R_2	hub and tip radii
t	time
U_r, U_a	inlet and outlet velocities of a stator vane
U_0	reference freestream velocity
V	vane number
v_x	acoustic axial velocity
$\mathbf{v}^h_K$	hydrodynamic velocity associated to the Kutta condition
(x,y)	axial and transverse Cartesian coordinates
$X=x/\beta$	scaled coordinate
α^j_s	scattered transverse wavenumbers of transmitted/reflected waves
$\beta=\sqrt{1-M_0^2}$	compressibility parameter
$\mathbf{\Gamma}$	pressure and axial-velocity vector
δ_1	displacement thickness
ϕ	acoustic potential
Φ_{pp}	wall-pressure power spectral density
Ψ_{pp}	dimensionless wall-pressure power spectral density
ρ_0	fluid density
σ	constant
ω	angular frequency
$\overline{\omega}$	dimensionless angular frequency
$\mathbf{\Omega_K}$	vorticity vector generated by the Kutta condition

Subscripts and superscripts

$(-)_0$	source-point coordinate
$(-)^0$	reference channel
$(-)_j$	incident wave index
$(-)_m$	channel mode index
$(-)_K$	Kutta-condition associated quantity
$(-)_s$	reflected or transmitted wave index
$(-)_{i,r,t,u,d}$	incident, reflected, transmitted, upstream and downstream potentials
$(-)^{\pm}$	for downstream/upstream wavenumbers

turbulent boundary layers developing on the blades and the vanes are scattered as sound at the trailing edges, also contributing to the broadband noise. Finally secondary flows such as tip-leakage vortices generate their own unsteadiness and associated noise. The aforementioned trailing-edge noise sources are not blade-to-blade or vane-to-vane correlated, which means that the sound is generated in the same way as for an isolated airfoil. But the sound radiation away from the trailing-edge area is more or less dramatically restructured by multiple scattering on adjacent blades/vanes. This restructuration referred to as the cascade effect is the main motivation of the present work. The emphasis is on the trailing-edge noise of the stator, though the same approach could be transposed to a rotor as explained later on.

In most architectures such as depicted in Fig. 1 the outlet guide vanes are moderately cambered and staggered at leading edge, and nearly parallel to the axis at trailing edge. Indeed they are designed to restore an axial mean flow of speed U_a from the incident swirl produced by the rotor and emphasized by the oblique relative speed of magnitude U_r in the figure. As a result the vanes have a large overlap and can be viewed from downstream as an array of parallel and zero-stagger plates. When trying to reproduce aeroacoustic phenomena using analytical approaches, the cascade effect of the stator appears as a key feature to deal with. The cascade effect is not only involved in the sound generation process but also when the sound generated by the rotor blades is transmitted downstream in the exhaust duct, especially for stators with a large number of vanes and quite large hub-to-tip ratios R_1/R_2. Many studies contributed to the development of analytical or semi-analytical cascade response functions for sound generation or transmission in blade rows [2–24]. The approach developed by Glegg [15] and later improved by Posson et al. [23] is selected here for the discussion. But only wake-interaction noise or turbulence-impingement noise is generally considered. The cascade effect on the trailing-edge noise mechanism is more difficult to formulate even though an attractive and elegant approach has been proposed by Glegg and Jochault [25]. The issue is that trailing-edge noise sources are localized and poorly correlated, which makes them difficult to describe in a cascade context. Though only outlet guide vanes are considered in the present work, the same would hold for the blades of a rotor except that most often the overlap is smaller, at least in the tip region of the blades, and that the number of blades is also smaller. Furthermore the stagger angle of the blades is quite large. The case of the stator is chosen here because the