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# Constrained Spectral Conditioning for spatial sound level estimation

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ABSTRACT

Microphone arrays are utilized in aeroacoustic testing to spatially map the sound emitted from an article under study. Whereas a single microphone allows only the total sound level to be estimated at the measurement location, an array permits differentiation between the contributions of distinct components. The accuracy of these spatial sound estimates produced by post-processing the array outputs is continuously being improved. One way of increasing the estimation accuracy is to filter the array outputs before they become inputs to a post-processor. This work presents a constrained method of linear filtering for microphone arrays which minimizes the total signal present on the array channels while preserving the signal from a targeted spatial location. Thus, each single-channel, filtered output for a given targeted location estimates only the signal from that location, even when multiple and/or distributed sources have been measured simultaneously. The method is based on Conditioned Spectral Analysis and modifies the Wiener–Hopf equation in a manner similar to the Generalized Sidelobe Canceller. This modified form of Conditioned Spectral Analysis is embedded within an iterative loop and termed Constrained Spectral Conditioning. Linear constraints are derived which prevent the cancellation of targeted signal due to random statistical error as well as location error in the sensor and/or source positions. The increased spatial mapping accuracy of Constrained Spectral Conditioning is shown for a simulated dataset of point sources which vary in strength. An experimental point source is used to validate the efficacy of the constraints which yield preservation of the targeted signal at the expense of reduced filtering ability. The beamforming results of a cold, supersonic jet demonstrate the qualitative and quantitative improvement obtained when using this technique to map a spatially-distributed, complex, and possibly coherent sound source.



## 1. Introduction

Arrays of microphones are used as spatial and temporal filters for the pressure signals which they measure. Processing of the array outputs yields spatial information of the sound source being studied which is not available when the microphones

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**Nomenclature**

*Latin*

| | |
|---|---|
| $\hat{\varnothing}_{\mathbf{mm}'}$ | Measured cross-spectral phase between $\mathbf{m}$ and $\mathbf{m}'$ |
| $\varnothing_{\mathbf{sm}}$ | Phase between $\mathbf{s}$ and $\mathbf{m}$ |
| $\varnothing_{\mathbf{s},\mathbf{mm}'}$ | Relative phase difference between $\mathbf{m}$ and $\mathbf{m}'$ due to source at $\mathbf{s}$ |
| $c$ | Speed of sound in propagation medium |
| $ch_{\mathbf{m}}$ | Pressure time record of microphone $\mathbf{m}$ |
| $CH_{\mathbf{m}}$ | Fourier transform of $ch_{\mathbf{m}}$ |
| $\mathbf{e_s}$ | Steering vector matrix for all $\mathbf{M}$ for location $\mathbf{s}$ |
| $\mathbf{e_{sm}}$ | Steering vector from $\mathbf{s}$ to $\mathbf{m}$ |
| $f$ | Frequency |
| $Fs$ | Sampling frequency |
| $\hat{\mathbf{G}}_{\mathbf{mm}'}$ | Measured cross-spectrum between $\mathbf{m}$ and $\mathbf{m}'$ |
| $G_{\mathbf{s},\mathbf{mm}}$ | Autospectrum induced on $\mathbf{m}$ from source $\mathbf{s}$ |
| $K$ | Number of data blocks that $ch_{\mathbf{m}}$ is divided into |
| $\mathbf{m}$ | Euclidean position vector of a microphone |
| $\mathbf{m}'_{\mathbf{op}}$ | Optimum reference microphone |
| $M$ | Total number of microphones in array |
| $\mathbf{M}$ | Total set of $\mathbf{m}$ |
| $r_{\mathbf{sm}}$ | Euclidean distance between spatial location $\mathbf{s}$ and microphone $\mathbf{m}$ |
| $\mathbf{s}$ | Euclidean position vector of scanning location (targeted point in space) |
| $\mathbf{s}_0$ | Euclidean position vector of a source |
| $\mathbf{s}'$ | Euclidean position vector of an intra-volume grid point for $\mathbf{s}$ |
| $S$ | Total number of sources in a measured field |
| $\mathbf{S}$ | Total set of $\mathbf{s}$ |
| $T$ | Length (in samples) of data block obtained from $ch_{\mathbf{m}}$ |
| $\mathbf{W_b}$ | Blocking/filtering weight term |

*Greek*

| | |
|---|---|
| $\hat{\gamma}^2_{\mathbf{mm}'}$ | Measured ordinary coherence between $\mathbf{m}$ and $\mathbf{m}'$ |
| $\Delta x$ | Euclidean distance between scanning grid points in the $x$-direction |
| $\Delta x'$ | Euclidean distance between scanning grid intra-grid points in the $x$-direction |
| $\Delta x_{\mathbf{s}}$ | Euclidean source dimension in the $x$-direction |
| $\varepsilon$ | Normalized error |
| $\sigma$ | Standard deviation |

*Abbreviations*

| | |
|---|---|
| CSC | Constrained Spectral Conditioning |
| CSM | Cross-Spectral Matrix |
| FDBF | Frequency-Domain Beamforming |
| RE | Random Error |
| SD | Standard Deviation |
| SVE | Steering Vector Error |

*Subscripts*

| | |
|---|---|
| **op** | Optimum |

*Superscripts*

| | |
|---|---|
| * | Complex conjugation |
| \widehat | Measured value (over variable) |
| † | Conjugate transposition |
| ′ | Different value of the same variable which does not carry this superscript |
| it | Iteration |

are used independently. However, the accuracy of the spatial mapping obtained is still being improved upon in the aeroacoustic community, and many separate efforts have been made to develop algorithms which post-process the array data to more accurately map the measured sound field [1–12]. One of the simplest and most commonly used processing formulations for synthesizing the sensor output data in order to obtain spatial information about the measured field is data-independent beamforming [13], which has been widely used in aeroacoustic testing since 1975 [14–20]. Optimal forms of beamforming (e.g., the Generalized Sidelobe Canceller (GSC, [21,22]) and Minimum Variance Distortionless Response [23]) which depend on the measured data (data-dependent) improve upon the data-independent formulation when properly constrained [24,25]. However, due to their nonlinear formulation and output uncertainty, only a few examples of their usage exist in the aeroacoustic literature [6,26–28].

Most optimal, data-dependent beamformers produce synthesized outputs, i.e., sound level estimation at targeted locations in the field is only available once the individual channel data are (manipulated and) summed. An exception can be found in [29] which provides a reformulation of the GSC that operates on the single-channel data in the time domain before array synthesis. The work presented in this paper can be viewed as a frequency-domain version of the technique from [29], and uses a more intuitive method of preventing cancellation of the targeted signal in the filtering process. This processing scheme linearly filters single-channel, frequency-domain data obtained from microphone array measurements in the manner of an optimal, data-dependent beamformer. The filtering preserves signal from user-defined spatial locations by leveraging the orthogonality between the targeted signal and all others present on the channels. Thus, single-channel outputs are produced at each targeted location which estimate only the signal from that location, even when multiple and/or distributed sources have been measured simultaneously. These single-channel, filtered outputs are available for further processing, examples being data-independent or -dependent beamforming [13], subspace methods [6,9,28], deconvolution [1–4,6–8,10–12], decomposition [5,9,30,31], or single-channel processing which does not employ array synthesis [32].

Section 2 defines the filtering equations used and their geometrical formulation for the preservation of targeted signal while minimizing the total signal on the channels. As the filtering requires other array channels which are used as