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Simulation of non-stationary wind velocity field on bridges based on Taylor series

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ABSTRACT

The simulation of non-stationary wind velocity field based on the spectral representation method often requires significant computational efforts due to the summation of trigonometric functions usually involved in the simulation procedure. Some techniques which make use of FFT algorithm have been developed but most of these techniques deal with seismic ground motions. Limited effort has been devoted to the simulation of non-stationary wind velocity. Therefore, in this paper, a spectral-representation-based technique which takes advantage of FFT algorithm is proposed by combining Cholesky decomposition and Taylor series expansion. The approach consists of locating and expanding the time and frequency non separable part of the decomposed evolutionary power spectral density function by mean of Taylor series expansion to allow the application of the FFT algorithm. Samples of non-stationary wind velocity can be then generated through multiple executions of the FFT algorithm once the Taylor series expansion is successful. The present approach, which is primarily developed for the simulation of non-stationary wind velocity on long-span cable supported bridges, is very efficient since the summation of the trigonometric functions can be carried out through FFT algorithm which is well known for its higher efficiency. The approach was further improved by reformulating the simulation formulas where the order in which the summation operations are executed is imposed.

1. Introduction

Modern long-span cable supported bridges may experience considerable wind-induced vibration due to their structural flexibility. The aerodynamic study of these kind of bridges requires suitable fluctuating wind velocity time-history which can be numerically generated by Monte Carlo simulation (Spanos and Zeldin, 1998; Kareem, 2008) which in turn may be achieved by frequency domain approaches such as linear filtering approaches (Li and Kareem, 1990; Deodatis and Shinozuka, 1988; Spanos and Mignolet, 1989) or by spectral representation (SR) method (Chen and Letchford, 2004; Shinozuka and Jan, 1972; Grigoriu, 1993).The spectral representation (SR) method is preferred for its higher accuracy. However, for the simulation of fluctuating wind velocity time-history on long-span cable supported bridges where thousands of long duration wind processes are required, the spectral representation method may become computationally expensive in terms of time and memory consumption. The SR method can be divided into two main phases. The first phase (phase I) consists of the decomposition of the power spectral density (PSD) matrix while the second phase (phase II) consists of the summation of trigonometric functions. Both phases have higher demand in terms of computational resources. The improvement of any of the two phases has significant impact on the overall simulation algorithm. In the last decades, several techniques have been developed to improve the efficiency of the SR method for the simulation of fluctuating wing velocity field on long-span bridges.

For long span cable-stayed bridge, Li et al. (2004) introduced a simplification approach to improve the computational complexity of the phase I (decomposition of the PSD matrix) by treating a three-dimensional wind velocity field as a group of one dimensional wind velocity fields. A highly efficient technique was introduced by Yang et al. (1997) and was later improved by Cao et al. (2000) for the simulation of wind velocity field on long-span bridge decks. They established a closed-form expression for the Cholesky decomposition of the PSD matrix based on the assumption that the simulation points are uniformly distributed at the same height along the bridge deck axis. Recently, similar approach has been proposed by Togbenou et al. (2016) for the

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simulation of wind velocity field on bridge towers to improve the efficiency of the phase I. Several approximation methods such as the proper orthogonal decomposition (POD) approach (Holmes et al., 1997; Rathinam and Petzold, 2003; Di Paola and Gullo, 2001; Solari and Carassale, 2000; Chen and Kareem, 2005) and interpolation methods (Ding et al., 2006; Gao et al., 2012; and Carassale and Solari, 2006) have been developed for the phase I (decomposition of the PSD matrix). It can be noticed that most of the techniques that were developed concern the improvement of the phase I. Few research has been conducted on the phase II (summation of trigonometric functions).

The generation of fluctuating wind velocity samples by direct summation of trigonometric series requires significant computation effort. Yang (1972) showed that the summation of the trigonometric series can be efficiently carried out by applying the Fast Fourier Transform (FFT) algorithm, which significantly reduces the computation effort (Chen et al., 2014; Deodatis, 1996b). Unfortunately, the FFT algorithm was limited to the simulation of stationary processes. The spectrum associated with non-stationary processes is time-dependent and therefore does not allow the direct application of the FFT algorithm. The application of the FFT algorithm was later extended to the simulation of non-stationary processes by Li and Kareem (1991) who suggested the decomposition of the evolutionary power spectra in terms of trigonometric or polynomial expansion based on stochastic decomposition which allowed the application of the FFT algorithm for non-stationary processes. Later, a spectral-representation-based approach was proposed in (Deodatis, 1996a) for the simulation of non-stationary, multivariate random process with prescribed evolutionary power. Although the approach did not address the issues related to the inapplicability of the FFT algorithm, however, more information on how to apply the spectral representation method for the simulation of non-stationary process were provided. Without the help of the FFT algorithm, the simulation of non-stationary stochastic process becomes very difficult especially when a large number of non-stationary processes has to be simulated. Recently, Huang (2015) proposed a simulation method in which the FFT algorithm can be used by combining Cholesky decomposition and the POD approach. With this technique which is a spectral-representation-based technique, the evolutionary power spectral density (EPSD) matrix is first decomposed based on Cholesky decomposition. Then, the resulted decomposed spectrum is expanded into a set of separate functions of time and frequency based on the POD approach. In terms of computation speed, the approach appears to be better than the one proposed in Li and Kareem (1991) due to the reduced number of items required for the POD. The FFT-based approach of Li and Kareem (1991) and that of Huang (2015) were primarily developed for the simulation of non-stationary seismic ground motions and their application to the simulation of non-stationary wind fields may not be straightforward due to the difference in spectrum shape. Therefore, this paper aims at proposing a simple and easy to apply technique, which provides a clear and comprehensive procedure for the application of the main idea of spectrum separation used in the FFT-based approaches of Li and Kareem (1991) and Huang (2015) for the simulation of non-stationary fluctuating wind velocity field for long-span cable supported bridges. The wind spectra that are involved in the study of the interaction between bridges and wind belong to a special class of spectra from which evolutionary spectra can be constructed and separated easily without the need of the stochastic decomposition and the POD approaches.

In this paper, a modified FFT-based spectral representation method is proposed for the simulation of non-stationary wind velocity fluctuation for long-span cable supported bridges by combining Cholesky decomposition and Taylor series expansion. The approach consists of a spectral representation method where the frequency and time non-separable part of the decomposed evolutionary power spectral density (EPSD) function is extracted and expanded through Taylor series expansion. Once the expansion is successful, the decomposed EPSD can be regarded as the weighted summation of frequency-dependent functions with time-dependent weights. In order words, the non-separable time and frequency PSD is converted into a set of separate functions of frequency and time. Based on the expanded decomposed evolutionary spectra, the summation operations involved in the phase II of the spectral representation method can be efficiently carried out through multiple executions of the FFT algorithm. The method was further improved by modifying the order in which the summation operations are conducted and improved simulation formulas are provided. The paper consists of four sections. The first section provides a review on the related existing works and introduces the current new contribution while the last section summarizes the entire study. The second section explains the proposed simulation procedure. A numerical example involving the simulation of non-stationary fluctuating wind velocity field along the deck of a cable supported bridge is performed in the section three to demonstrate the accuracy and the efficiency of the proposed simulation procedure.

2. Non-stationary wind velocity field simulation methods

2.1. Simulation of wind velocity by spectral representation method

Let's assume that $\mathrm{S}(\omega,\mathrm{t})$ is the time and frequency non-separable evolutionary power spectral density matrix of a stochastic zero-mean non-stationary wind velocity field $x(t)$. $\mathrm{S}(\omega,\mathrm{t})$ can be expressed as follows:

$$\mathrm{S}(\omega,\mathrm{t})=\begin{bmatrix}\mathrm{S}_{11}(\omega,\mathrm{t}) & \mathrm{S}_{12}(\omega,\mathrm{t}) & \cdots & \mathrm{S}_{1n}(\omega,\mathrm{t})\\ \mathrm{S}_{21}(\omega,\mathrm{t}) & \mathrm{S}_{22}(\omega,\mathrm{t}) & \cdots & \mathrm{S}_{2n}(\omega,\mathrm{t})\\ \vdots & \vdots & \ddots & \vdots\\ \mathrm{S}_{n1}(\omega,\mathrm{t}) & \mathrm{S}_{n2}(\omega,\mathrm{t}) & \cdots & \mathrm{S}_{nn}(\omega,\mathrm{t})\end{bmatrix} \tag{1}$$

$$\mathrm{S}_{jk}(\omega,\mathrm{t})=\sqrt{\mathrm{S}_{jj}(\omega,\mathrm{t})\mathrm{S}_{kk}(\omega,\mathrm{t})}\gamma_{jk}(\omega) \tag{2}$$

where ω is the circular frequency; $\mathrm{S}_{jj}(\omega,\mathrm{t})$ and $\mathrm{S}_{kk}(\omega,\mathrm{t})(j,k=1,2,\cdots n)$ are the auto spectral density functions of the components $x_j(t)$ and $x_k(t)$, respectively; $\gamma_{jk}(\omega)$ is the coherence function. The EPSD matrix $\mathrm{S}(\omega,\mathrm{t})$ can be decomposed in the following form (Gao et al., 2012; Li et al., 2004):

$$\mathrm{S}(\omega,\mathrm{t})=\mathrm{D}(\omega,\mathrm{t})\Gamma(\omega)\mathrm{D}^{\mathrm{T}^*}(\omega,\mathrm{t}) \tag{3}$$

where

$$\mathrm{D}(\omega,\mathrm{t})=\mathrm{diag}\left[\sqrt{\mathrm{S}_{11}(\omega,\mathrm{t})},\sqrt{\mathrm{S}_{22}(\omega,\mathrm{t})},\cdots,\sqrt{\mathrm{S}_{nn}(\omega,\mathrm{t})}\right] \tag{4}$$

T denotes transpose and * denotes the complex conjugate. $\Gamma(\omega)$ is the lagged coherence matrix:

$$\Gamma(\omega)=\begin{bmatrix}1 & \gamma_{12}(\omega) & \cdots & \gamma_{1n}(\omega)\\ \gamma_{21}(\omega) & 1 & \cdots & \gamma_{2n}(\omega)\\ \vdots & \vdots & \ddots & \vdots\\ \gamma_{n1}(\omega) & \gamma_{n2}(\omega) & \cdots & 1\end{bmatrix} \tag{5}$$

Both $\mathrm{S}(\omega,\mathrm{t})$ and $\Gamma(\omega)$ are hermitian matrices with positive definite properties and therefore can be decomposed in the following form:

$$\mathrm{S}(\omega,\mathrm{t})=\mathrm{D}(\omega,\mathrm{t})\mathrm{L}(\omega)\mathrm{L}^{\mathrm{T}^*}(\omega)\mathrm{D}^{\mathrm{T}^*}(\omega,\mathrm{t})=\mathrm{H}(\omega,\mathrm{t})\mathrm{H}^{\mathrm{T}^*}(\omega,\mathrm{t}) \tag{6}$$

where $\mathrm{H}(\omega,\mathrm{t})$ and $\mathrm{L}(\omega)$ are lower triangular matrices which can be computed based on the Cholesky decomposition. The computation of $\mathrm{H}(\omega,\mathrm{t})$ through the Cholesky decomposition will require higher computation time due the fact that the decomposition has to be executed for each frequency increment and time instant. Therefore, the decomposed lagged coherence matrix $\mathrm{L}(\omega)$ is computed instead. By referring to Eq. (6), the elements of the matrix $\mathrm{H}(\omega,\mathrm{t})$ can be estimated as follows:

$$\mathrm{H}_{jk}(\omega,\mathrm{t})=\sqrt{\mathrm{S}_{jj}(\omega,\mathrm{t})}\beta_{jk}(\omega) \tag{7}$$