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Wind modelling with nested Markov chains

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ABSTRACT

Markov chains (MCs) are statistical models used in many applications to model wind speed. Their main feature is the ability to represent both the statistical and temporal characteristics of the modelled wind speed data. However, MCs are not able to capture wind characteristics at high frequencies, and, by definition, in an MC the dependence on events far in the past is lost. This is reflected by a poor match of autocorrelation function of recorded data and artificially generated time series. This study presents a new method for generating artificial wind speed time series. This method is based on nested Markov chains (NMCs), which are an extension of MC models, where each state in the state space can be seen as a self-contained MC. The approach is designed to be flexible, so that the number and distribution of NMC states can be adjusted according to user requirements for model accuracy and computational efficiency. The model is tested on two datasets recorded in two UK locations, one onshore and one offshore. Results indicate that NMCs are able to capture the temporal self-dependence of wind speed data better than MCs, as shown by the better match of the autocorrelation functions of recorded and artificially generated time series.

1. Introduction

The efficient analysis and exploitation of wind energy resources requires models for wind speed at different time scales. The aim of these models is not to forecast the actual wind speed at a certain time, but to generate artificial wind time series that can realistically represent a possible chain of events, i.e. series of wind speeds with a pre-set resolution. Depending on the application, there are some aspects of this "realism" that might be more important than others. This is the case, for instance, of extreme events modelling (Lennard, 2014), or investigation of daily patterns in wind energy production (Scholz et al., 2014), or estimation of total annual energy outputs of wind farms (Hayes and Djokic, 2013b; Hayes et al., 2011, 2012).

Different methods for wind speed modelling have been proposed such as autoregressive moving average (ARMA) models (Kennedy and Rogers, 2003) and Markov chain (MC) models (Jones and Lorenz, 1986). More sophisticated and accurate methods, which may for instance use the knowledge of other quantities such as pressure and temperature, have been developed (e.g. Bitner-Gregersen et al., 2014), but those methods are more computationally demanding and not suitable for applications where a limited amount of data is available. Despite their simplicity, MCs are able to model the wind time dependence characteristics because they are based on the idea that the probability distribution for the wind at the next time step depends on the current wind state. Other models, such as ARMA, are not able to capture this probability dependence. Therefore, although the need for forecasting has driven academic research to develop better models, the simplicity of MCs makes them a valuable tool as shown by their use in many recent studies. For example, when wind influences a series of decisions that have to be based on current observation, MCs are particularly suited for their property of memory loss (Al-Sabban et al., 2013). Similarly, MCs have been used to model wind turbines when focusing on component failure, that has properties that are independent from the past history (Sunder Selwyn and Kesavan, 2013), or in sailing strategy, where decisions taken at one time step need to be based on the expected wind behaviour at the following time step(s) (Tagliaferri et al., 2014).

However, MCs are not able to capture wind characteristics at high frequencies, but also, by definition, in an MC the dependence on events far in the past is lost. This is reflected in a general good agreement of statistical quantities such as mean and variance, but in a poor modelling of autocorrelation function and power spectral density. A recent study by Brokish and Kirtley (2009) underlines the appeal of MCs for wind modelling in terms of correct

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representation of cumulative distribution function (CDF), but also shows the unsuitability of the model for time steps smaller than 15 min using a convincing example of storage underestimation.

In order to improve the accuracy and the autocorrelation of standard MCs, semi-Markov models have been used, where the time step is not fixed, but it is a random variable that can have any distribution, and the time spent in one state affects the transition probability distribution (D'Amico et al., 2014). In D'Amico et al. (2013), it is shown how semi-Markov processes with memory exhibits a better autocorrelation agreement than conventional MCs. This is due to the ability of this model to keep memory of past transitions through an auxiliary random process representing the moving average of the wind speed.

Some authors (e.g. Shamshad et al., 2005) have also applied second or third order MCs, where time steps are again fixed, and the probability distribution for the next state is dependent not just on the current state, but also on the previous states. Unfortunately, higher order MCs are more computationally demanding, as for instance a third order 32-state MC requires 32,768 state transition probabilities. Therefore, the key advantage of using MCs instead of a more sophisticated method is lost.

In order to improve the MC accuracy and to better model the time correlation at small time steps without an excessive increase of the computational time, we propose the use of nested Markov chains (NMCs) for wind modelling, which is previously considered in the context of "smart grid" analysis in Hayes and Djokic (2013a). With a similar approach to the one presented in D'Amico et al. (2013), we define a model based on MC but with the additional property of keeping a form of memory of past transitions.

The paper is organised as follows: in the Method, we present the principles of MCs and NMCs, how these models are used to forecast the wind speed, and the criteria used to evaluate the results. In the Results, we compare different artificial time series generated with MCs and NMCs with original recorded data. Concluding remarks are summarised in the Conclusions.

2. Method

2.1. Markov chains

In this section we define MCs and their basic properties. A complete description of MC is out of the scope of this paper and can be found for instance in Norris (1998).

Let $X_0, X_1, X_2, \ldots$ be the stochastic process representing the wind speed. The subscript represents a discrete time step (seconds in this work) and the random variables X_i can assume values on a discrete set $\mathcal{S} = \{s_1, \ldots, s_N\}$, which is called state space. In the present study, the states $s_1, s_2, \ldots$ are intervals of possible wind speeds, and each interval is identified by its central point. The states are classified in Table 1. With this notation, the wind speed is represented as a time series, or a stochastic process, where, for instance, the events "$X_0 = s_3$" and "$X_4 = s_8$" mean that the wind speed at time 0 (or initial time) is in the interval s_3 and that the wind speed a time $t = 4$ s is in the interval s_8 respectively. For simplicity, when generating an output time series, we consider just the central point of the interval defining the state. This means that the event "the wind speed value is in the range $[a, b]$" becomes the event "the wind speed is $(b + a)/2$". The choice of having wider intervals grouped in the same state for higher wind speeds is justified by the infrequent occurrence of those wind speeds. This results in a trade-off between the number of states and how accurately the higher, more infrequent wind speeds are modelled. However, the occurrences of infrequent wind speeds, and therefore the choice of interval widths, depends on the available dataset (specifically on its length). The Markov property for the process $\{X_k\}_{k \geq 0}$ asserts that the probability distribution at time n is dependent on the state at time $n - 1$, but independent from what happened before. This property is also referred to as *memory loss*, and is formulated by the following equation:

$$\mathbb{P}\{X_n = s_j | X_{n-1} = s_i, X_{n-2} = s_{i_{n-2}}, \ldots, X_0 = s_{i_0}\} = \mathbb{P}\{X_n = s_j | X_{n-1} = s_i\} = p_{ij} \quad (1)$$

where $s_i, s_j, s_{i_k} \in \mathcal{S}$. Fig. 1 shows a common way of representing MC. The process "*jumps*" from one state to the next according to the probabilities associated to the arrows. It is clear from the representation that the transition probabilities depend on the current state, but not on the previous ones. The transition probabilities are naturally represented in a transition matrix $P = \{p_{ij}\}$, where the elements of the matrix, p_{ij}, are the probabilities defined in Eq. (1). The ith row of the matrix P represents the discrete probability distribution for the next state when the current state is i. The probability distribution for the initial state X_0, or *initial distribution* is conventionally represented as a column vector P_0, where the element p_i^0 is defined in Eq. (2), or the initial state could be arbitrarily selected to start the process (for instance, as the mode or median value from the dataset):

$$p_i^0 = \mathbb{P}\{X_0 = s_i\} \quad (2)$$

State space, transition matrix and initial distributions uniquely define an MC process.

2.2. Nested Markov chains

In the NMC approach, the wind series is built using an auxiliary MC. Let T and t be two different time steps, where T is a multiple of t. For instance, throughout this paper t=1 s. Let $S = s_1, \ldots s_N$ be a finite state space. We define $Y_0, Y_1, \ldots$ the MC on the space state S, representing the average wind speed over a period of length T with transition matrix P. In the following, this process will be referred to as the *outer* MC. We generate a sequences of wind speed time series of duration T with time step t using the transition matrix P^{Y_i}, where the element P_{ij}^k represents the probability that the wind at instant n is in state s_j given the event that at time $n - 1$ it was in state s_i when the average over the period T is s_k. Those models will be referred to as *inner* MC. The output process is the sequence of realisations of the inner MC, i.e. one series if inner states with step t for each state of the outer MC. Fig. 2 shows a graphical representation of the relationship between inner and outer MC.

The output process now does not strictly follow the Markov property, because the probability distribution for time n does not depend only on the state at time $n - 1$, but also on what happened in the previous hour. However, if we consider the process within one hour, this is an MC. Also the outer process is an MC. In fact, an

Table 1
State spaces.

State	s_1	s_2	…	s_{26}	s_{27}	s_{28}	s_{29}	s_{30}	s_{31}	s_{32}
Interval (m/s)	0–1	1–2	…	25–26	26–28	28–31	31–34	34–39	39–43	43–54
Output (m/s)	0.5	1.5	…	25.5	27	29.5	32.5	36.5	41	48.5