J. Wind Eng. Ind. Aerodyn. 157 (2016) 125–139

Contents lists available at ScienceDirect

Journal of Wind Engineering and Industrial Aerodynamics

journal homepage: www.elsevier.com/locate/jweia

The extended Davenport peak factor as an extreme-value estimation method for linear combinations of correlated non-Gaussian random variables

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ARTICLE INFO

Article history:
Received 31 January 2016
Received in revised form
30 June 2016
Accepted 23 July 2016

Keywords:
Extreme value distribution
Peak factor
Combination rule
Non-Gaussian process
Parabolic trough

ABSTRACT

This article develops a method for obtaining the distribution function of the extreme values that result from the linear combination of a finite number of non-Gaussian random variables that come from a common agent within a period of time. The approach incorporates the variability of both the components and their interactions, identifies the conditions for the arrival of extreme values and bounds the uncertainty in the results. The outcome of the method is an "extended Davenport peak factor" (EDPF) that relates the descriptors of the resultant, the parameters of the distribution function of the peaks, and the probability of not exceeding a given threshold.

The method was applied to the analysis of the extreme wind actions on a parabolic trough. The model parameters were expressed in terms of 3 magnitudes which characterize the spatial (correlation), temporal (frequency of peaks) and intrinsic (variance) variability of the actions. The results are consistent with methods that consider the variability of the components and the variability that is introduced by their combination. Nonetheless, the methods that do not incorporate this information result in peak factors that are an average of 13% lower. Also, the methods for Gaussian variables do not capture the variability between the case studies.

1. Introduction

The characterization of the total response of a structural system that is subjected to natural agents requires a precise description of the load actions. These actions have spatial and temporal variabilities due to the intrinsic variability of the agents and their interactions with the geometric and mechanical properties of the system. For example, loads in a solar collector are mainly caused by the wind acting over the mirrors, and are transmitted to the foundation through the support structure. In this example, the agent is the wind and can be described by its speed, direction, turbulence intensity, and spectral density function. The actions on the system are quantified as the net pressure on each mirror, and the system response is the total load on the structure, which is calculated as the linear superposition of the actions.

Civil engineering often relies on designs that require the identification of the extreme value of a variable (agent, action or response) which is combination of several components and whose exceedance probability within a period T must be below a given value. This requires considering the variability that is introduced by the processes themselves (Gaussian or not) and by their interactions according to the number of components, the manner in which they combine, and their dependence relations. The peak factor g is a common tool that is useful to express the largest value of a random variable Y in T based on its statistical and spectral properties. This parameter is defined in Eq. (1)

$$g(T)=\left(E\left[Y_{max,T}\right]-\mu_Y\right)/\sigma_Y \tag{1}$$

where $Y_{max,T}$ is the variable of extreme values of Y in T, $E\left[Y_{max,T}\right]$ is its expected value, μ_Y is the mean value of Y, and σ_Y is its standard deviation.

For a single random variable, Davenport (1964) demonstrated that the distribution of the extreme values in T asymptotically follows a Fisher–Tippett Type I distribution function under the following assumptions: (1) the process is Gaussian; (2) the up-crossing of the variable through a sufficiently high threshold is a Poisson event; and (3) the number of relative maxima in T is large.

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http://dx.doi.org/10.1016/j.jweia.2016.07.014

Nomenclature

N_s	Number of simultaneous peaks over a time interval Δt.
X_i	Instantaneous value of the ith component of the resulting action.
$X_{\Delta ti}$	Average value of X_i over a regular time interval Δt.
X_{imaxT}	Extreme value of $X_{\Delta ti}$ during a period of T.
X_{p_i}	Peak value of $X_{\Delta ti}$ conditioned by the occurrence of a peak in the resultant.
X_{np_i}	Non-peak value of $X_{\Delta ti}$ conditioned by the occurrence of a peak in the resultant.
$\mathcal{X}_p$	Value of the sum of X_{p_i} over a time interval Δt.
$\mathcal{X}_{np}$	Value of the sum of X_{np_i} over a time interval Δt.
Y	Instant resultant value of the combination of N other actions X_i.
$Y_{\Delta t}$	Value of Y averaged at regular time intervals Δt.
$Y_{max,T}$	Extreme value of $Y_{\Delta t}$ during a period of T.
Y_p	Peak value of $Y_{\Delta t}$.
Y_{np}	Non-peak value of $Y_{\Delta t}$.
$Cov(U, V)$	Covariance operator between the variables U and V.
$E[U]$	Expected value operator of the variable U.
N	Number of actions on the system.
T	Duration of the calculation state.
$Var(U)$	Variance operator of the variable U.
g	Peak factor (Davenport, 1964).
$g_{u_{NE}}$	Extended Davenport peak factor associated with a non-exceedance probability of u_{NE}.
m_i	Spectral moment of order i.
t	Time
$\Gamma(x)$	Gamma function of x.
γ	Euler-Mascheroni constant ($\gamma \approx 0.5772$).
μ_U	Mean value of the random variable U.
ν_U	Mean peak frequency of the variable U.
$\rho_{U,V}$	Correlation coefficient between the variables U and V.
σ_U	Standard deviation of the random variable U.

This leads to the well-known expression for the peak factor (Eq. (2))

$$g(T) = \sqrt{2\cdot\ln\left(\nu_0^+ T\right)} + \gamma / \sqrt{2\cdot\ln\left(\nu_0^+ T\right)}, \tag{2}$$

where ν_0^+ is the mean upcrossing rate through the mean value of the variable, and γ=0. 5772 is the Euler constant. This relation has been widely used in wind engineering because of its satisfactory results for a large variety of conditions. Furthermore, this relation varies little over a wide range of $\nu_0^+ T$ values, which facilitates the characterization of reference magnitudes. For example, in the case of wind with a characteristic duration of $T = 10$ min and dominant frequencies of interest between 0.03 and 3.00 Hz, g is bounded within the interval 2.6–4.0.

Davenport's model has several limitations that affect the practice of engineering. Holmes (1985) and Kareem et al. (1995) showed that when the process is non-Gaussian, the result can be nonconservative. To overcome this shortcoming, Kareem and Zhao (1994) proposed including the third- and fourth-order central moments of the variable in the expression for the peak factor. The Kareem and Zhao (1994) approach has inherited a shortcoming of its underlying model, Hermite polynomial model by Winterstein (1988), which is well-known to be valid for the process with small/mild non-Gaussianity. Later revisions, that extend the reach of this method to a wider range of the skewness and kurtosis values, can be found in Kwon and Kareem (2011), Yang et al. (2013) or Peng et al. (2014) for softening process or in Ding and Chen (2014) for hardening process, among others. An alternative strategy (Sadek and Simiu, 2002; Huang et al., 2013) is based on the selection of an adequate distribution for the non-Gaussian variable followed by a translation process between the Gaussian and non-Gaussian spaces. In this approach, an appropriate distribution model should be considered to well represent the distribution tail.

Vanmarcke (1975) showed that the spectral bandwidth of the variable affects the performance of the model of Davenport (1964) and proposed alternative solutions. For a narrow-band process, the model does not consider the dependence between consecutive upcrossings. In contrast, for wide-band processes, the model does not consider the time that the signal remains above the threshold. Pillai and Tamura (2007) reformulated the expression for the peak factor by Kareem and Zhao (1994) to introduce the effect of the spectral bandwidth by means of the parameter of Cartwright and Longuet-Higgins (1956).

The determination of the extreme values of the combination of multiple variables is often required in the structural designs. The main difficulties become how to translate the uncertainties of the variables X_i and the uncertainties due to their interactions into the calculation of the extreme value distribution function $\mathcal{F}\left(y_{max,T}\right)$ of the resultant, and how to deal with the different time variability of the variables. A general approach must consider which variables affect the extreme value of Y ($Y_{max,T}$), which of them are simultaneous (that is, occur at the same instant), and what are the values that are compatibles (that is, the simultaneous values which joint probability of occurrence is not negligible). The coefficient of correlation, which express the degree of linear dependence between two variables, is commonly used to consider compatibility in a simple way.

The combination can be done by using probabilistic methods such as the Ferry-Borges and Castanheta (1971) method, the load coincident method (Wen, 1977) or the point-crossing method (Larrabee, 1981). The load coincident method and the point-crossing often become too complex for common practice (Melchers, 1999), especially when the number of variables is large. Furthermore, the use of the Ferry-Borges and Castanheta (1971) method demands restrictive requirements to the stochastic process. Thus, deterministic methods that combine the characteristic values of the extreme-value distributions of the variables, such as the Turkstra's rule (Turkstra, 1970) or the complete quadratic combination (CQC), are commonly used.

Turkstra (1970) assumed that the extreme value of the resultant arrives when one of the components also has its extreme value. For only two actions, Turkstra's rule is

$$y_{max,T} = max\left[x_{1\ max,T} + x_2^{m1};\, x_{2\ max,T} + x_1^{m2}\right], \tag{3}$$

where x_1^{m2} and x_2^{m1} are the companion values of $x_{2\ max,T}$ and $x_{1\ max,T}$, respectively. In practice, the latter two are replaced by characteristic values of X_1 and X_2 (usually their mean values or their mean values plus the standard deviation), whose choice is often difficult to justify (Naess, 1989). Naess and Røyset (2000) extend Turkstra's original model to incorporate the effect of correlations $\rho_{1,2}$ between the different components. Chen (2015) analyzed the original rule and the variant by Naess and Røyset (2000) and concluded that both can significantly underestimate or overestimate the extremes of resultant responses depending on the ratio and correlation coefficient of the response components.

The CQC is another commonly used method when the combination of variables is linear. For two random variables it yields Eq. (4), where $\bar{X}_{i,max,T}$ is the expected extreme value of X_i, and $\rho_{1,2}$ is the correlation coefficient between X_1 and X_2. The exceedance